Stock Market Comovements in Central Europe: Evidence from Asymmetric DCC Model

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Abstract

We examine time-varying stock market comovements in Central Europe employing the asymmetric dynamic conditional correlation multivariate GARCH model. Using daily data from 2001 to 2011, we find that the correlations among stock markets in Central Europe and between Central Europe vis-à-vis the euro area are strong. They increased over time, especially after the EU entry and remained largely at these levels during financial crisis. The stock markets exhibit asymmetry in the conditional variances and in the conditional correlations, to a certain extent, too, pointing to an importance of applying sufficiently flexible econometric framework. The conditional variances and correlations are positively related suggesting that the diversification benefits decrease disproportionally during volatile periods.

JEL-Classification: G01, G15

Keywords: stock market comovements, Central Europe, financial crisis

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1 Introduction

It has been well documented that stock market volatility increases more after negative shock than after a positive shock of the same size. This asymmetry in stock market volatility has been extensively examined within univariate GARCH models (Engle and Ng, 1993). Nevertheless, the evidence on asymmetry in the conditional correlations among stock markets is more limited but has gained importance with the global financial crisis characterized by a series of joint negative shocks and increased turbulence.

In this paper, we study the stock market comovements in three Central European countries, both among these countries as well as vis–à–vis the Western Europe. We apply the asymmetric dynamic conditional correlation (ADCC) model developed by Cappiello et al. (2006). This class of multivariate GARCH models might be well suited to examine stock market comovements during the global financial crisis, as stock markets are typically hit by common rather than idiosyncratic shocks. An application of ADCC model and the focus on the effect of financial crisis on stock market comovements should differentiate our research from a large body of literature on interdependence among different Central European markets (Kasch-Haroutounian and Price, 2001, Scheicher, 2001, Voronkova, 2004, Patev et al., 2006, Egert and Kocenda, 2007, Syriopulos, 2007, Gilmore et al., 2008, Wang and Moore, 2008, Savva and Aslanidis, 2010, Kocenda and Egert, 2011, Hanousek and Kocenda, 2011, Syllignakis and Kouretas, 2011 or Horvath and Petrovski, 2012). Despite that the body of previous literature is rather extensive, some important issues still lack consensus. For example, some studies detect the presence of a long–term relation among stock markets in Central Europe and Western Europe, while others conclude that such long–term relation does not exist.

Our research focuses on the largest Central European stock markets (namely, the Czech, Polish and Hungarian stock markets) in 2001–2011. Previous studies examining the interdependence among these markets rarely allowed for the asymmetry in the conditional variance and to our knowledge, never investigated the asymmetry in the conditional correlation dynamics. In fact, the evidence on the asymmetry in the conditional correlation dynamics among stock markets is limited even for developed countries.

Cappiello et al. (2006) emphasize that if correlations and volatilities in stock markets move in the same direction, the long–run risks are higher than they appear in the short–run. Clearly, the evaluation of long–run risks is particularly important during the financial crisis and using rolling stepwise regression, we investigate this issue for Central European stock markets. In addition, we also examine whether stock market comovements have changed during the crisis. On the one hand, the global nature of recent financial crisis might imply that the comovements should become stronger. On the other hand, Central European countries were hit rather unequally by the crisis. The Czech and Polish financial system remained largely stable and Poland even maintained relatively solid growth during this period. On the other hand, Hungary experienced some instability in the banking sector triggered by the interplay of exchange rate fluctuations and adverse balance sheets effects.
because of debt denominated in foreign currencies. In addition, Hungarian sovereign debt rating has been downgraded several times during the crisis. In consequence, this might decrease the correlations between Hungarian and Czech as well as Polish stock markets during the crisis. Therefore, it is not clear \textit{a priori}, which effect prevails.

Our results indicate that Central European stock markets exhibit asymmetry in the conditional variances but the asymmetry in the conditional correlations is less frequent. Therefore, the results point to an importance of applying appropriately flexible modelling framework to assess the stock market comovements accurately.

We find that stock market correlations increased over time. The increase in the correlations is observed both for the Central European stock markets among themselves as well as vis-à-vis the euro area. The stock market correlations become more volatile during the financial crisis and, on average, the correlations remained at its pre-crisis level but still higher than the values typical for the period before the EU entry. We also find that the stock market conditional volatility and correlation are positively related as in Cappiello et al. (2006).

This paper is organized as follows. Related literature is discussed in Section 2. Our data are described in Section 3. The econometric model is introduced in Section 4. The results are presented in Section 5. The concluding remarks are given in Section 6.
2 Related Literature

We focus on the studies examining the stock markets in Central Europe using multivariate GARCH models in this section. The discussion of other studies employing predominantly Granger causality tests and cointegration techniques is available in Horvath and Petrovski (2012).

Using daily data in 1994–1998, Kasch-Haroutounian and Price (2001) investigate the interdependence among four CEE stock markets (the Czech Republic, Poland, Hungary and Slovakia) employing two different multivariate GARCH approaches – the constant conditional correlation (CCC) and BEKK model. Using the CCC model, they find a positive and statistically significant conditional correlation coefficient between Czech and Hungarian stock markets (the value of 0.22), and between Hungarian and Polish stock markets (0.13). For the other pairs, correlations are very small and statistically non-significant. Moreover, applying the BEKK model, they detected only one unidirectional volatility spillover from Budapest stock market to Warsaw stock market.

Scheicher (2001) examines the comovements between three European emerging markets (the Czech Republic, Poland and Hungary) in 1995–1997, using a vector autoregression (VAR)–CCC model. The results indicate both regional and global spillovers in returns but only regional spillovers in volatilities. This suggests that global shocks are transmitted to the CEE stock markets through return rather than volatility shocks.

Using the CCC and smooth transition CC (STCC) models, Savva and Aslanidis (2010) investigate the stock market integration among five Central and Eastern European (CEE) countries (the Czech Republic, Poland, Hungary, Slovakia and Slovenia) vis–à–vis aggregate euro area market in 1997–2008. The largest CEE markets (namely, the Czech Republic, Poland and Hungary) exhibit higher correlations vis–à–vis the euro area as compared to Slovenia and Slovakia. They also find the Czech Republic, Poland and Hungary to be the most interconnected markets in the region. Furthermore, they find increasing correlations among the CEE markets, and between Polish, Slovenian and Czech markets vis–à–vis the euro area. The correlations for other stock market pairs are broadly stable in time. Interestingly, the increase in the correlations between CEEs and the euro area occurs much earlier than among the CEE markets itself suggesting the strong influence of euro area developments on Central Europe.

Using a DCC model, Wang and Moore (2008) examine the interdependence (and its drivers) between three major emerging markets (the Czech Republic, Poland and Hungary) vis–à–vis the aggregate euro area market. They find that financial crisis and the EU enlargement has substantially increased the correlations between CEE markets and the euro area market. On the other hand, the financial depth contributes to the higher degree of correlations. Monetary and macroeconomic developments are not found to influence the correlations.
Syllingnakis and Kouretas (2011) employ a DCC model for weekly data in 1997–2009 and investigate the stock market correlations between three major stock markets (the US, Germany and Russia) and the Central and Eastern Europe (the Czech Republic, Estonia, Hungary, Poland, Romania, Slovakia and Slovenia). They find that the stock market correlations increase over time and argue that this reduces the diversification benefits in the CEE markets. They suggest that the shift in the correlation coefficients can be mainly explained by a greater degree of financial openness, followed by an increased presence of foreign investors in the region, and finally the entry in the EU.

Using daily data in 2006–2011, Horvath and Petrovski (2012) analyze both Central (the Czech Republic, Hungary and Poland) and South Eastern European (Croatia, Macedonia and Serbia) stock markets and their correlations with the Western Europe. Using the BEKK–GARCH model, they analyze the linkages between CEE and SEE stock markets vis–à–vis euro area. Their results indicate a high degree of integration between CEEs and the euro area (the value of correlations fluctuates around 0.6) and a low degree of integration between SEEs and euro area (the correlations fluctuate around 0). Among the SEE markets, Croatia exhibits an upward trend in the stock market correlations. Finally, their results suggest that financial crisis did not change the degree of stock markets substantially.

Although most studies employ weekly or daily data, there are several contributions based on intraday data (Egert and Kocenda, 2007, Hanousek and Kocenda, 2011, and Kocenda and Egert, 2011). Using the DCC model, Kocenda and Egert (2011) examine the comovements between three developed (France, Germany and the United Kingdom) and three emerging stock markets (the Czech Republic, Poland and Hungary). They find very low correlations among the emerging markets (ranging from 0.02 to 0.05), and between emerging markets and developed ones (ranging from 0.01 to 0.03). This indicates that the speed of transmission of shocks from the Western Europe is rather within days rather than at the higher frequency. The correlations among the developed markets appear to be large, indicating the high degree of integration of these markets. They observe an increase in the correlations in the CEE markets beginning in the second half of 2004, which is likely to be a consequence of those three countries joining the European Union.

In summary, previous literature suggests that the stock market correlations between Central and Western Europe increased somewhat over time and the strong correlations among these markets are visible for the data at daily or weekly frequency rather than when using intraday data. We revisit these findings using more general multivariate GARCH model, the ADCC, and focus on the effect of financial crisis and the nature of interactions between stock market volatility and correlations.
3 Stock Markets in Central Europe

The data set comprises daily closing price indices of three CEE countries and euro area for the period from December 20, 2001 to October 31, 2011, a total of 2,533 observations. It consists of stock indices of the Czech Republic (PX), Hungary (BUX), Poland (WIG) and the euro area (STOXX50).\textsuperscript{1} The source of our data is Reuters Wealth Manager. Figure 1 presents the plot of stock market indices.

Figure 1: Stock Market Indices

The values of BUX and WIG are given on the left axis and the values of PX and STOXX50 are given on the right axis.

All the above mentioned price series $P_t$ are transformed by taking the log first–difference, resulting in the return series $r_t = \log \left( \frac{P_t}{P_{t-1}} \right)$ (see Figure 2). Table 1 gives the descriptive statistics and several basic statistical tests performed on the index returns. The null hypothesis of unit root is rejected at 5% significance level for all return series. Furthermore, the returns are negatively skewed (except STOXX50, which is slightly positively skewed) and leptokurtotic, indicating that they are not normally distributed. In addition, we have tested the presence of autocorrelation and ARCH effects in returns using Ljung–Box $Q$ and ARCH–LM tests. The null hypotheses of no autocorrelation and no ARCH effects are rejected for all the series at the 5% significance level. A significant autocorrelation in the returns and mainly in the squared returns (indicating the presence of ARCH effects) is also observed in the sample autocorrelation functions (ACF) and partial autocorrelation functions (PACF) of (squared) returns.\textsuperscript{2} All in all, the above–mentioned return series exhibit the standard features of a financial time series.

\textsuperscript{1} Slovak stock market is not examined given that its liquidity is not high.

\textsuperscript{2} These results are available upon request.
Table 1: Summary statistics

<table>
<thead>
<tr>
<th></th>
<th>BUX</th>
<th>PX</th>
<th>WIG</th>
<th>STOXX50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. Dev.</td>
<td>0.0169</td>
<td>0.0156</td>
<td>0.0134</td>
<td>0.0143</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.1243</td>
<td>-0.5846</td>
<td>-0.3704</td>
<td>0.0999</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>9.4</td>
<td>16.61</td>
<td>6.12</td>
<td>9.58</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.1265</td>
<td>-0.1619</td>
<td>-0.0829</td>
<td>-0.09</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.1318</td>
<td>0.1236</td>
<td>0.0608</td>
<td>0.1022</td>
</tr>
<tr>
<td>Jarque–Bera stat.</td>
<td>4.323</td>
<td>19.687</td>
<td>1.083</td>
<td>4.573</td>
</tr>
<tr>
<td>Q(8) stat.(^a)</td>
<td>48</td>
<td>44.87</td>
<td>25.65</td>
<td>66.42</td>
</tr>
<tr>
<td>ARCH–LM stat.(^b)</td>
<td>328.2</td>
<td>511.55</td>
<td>245.86</td>
<td>449.67</td>
</tr>
<tr>
<td>ADF stat.(^c)</td>
<td>-20.21</td>
<td>-20.71</td>
<td>-27.62</td>
<td>-23.6</td>
</tr>
</tbody>
</table>

\(^a\) Q stands for Ljung–Box Q test. \(^b\) 4 lags are used in ARCH–LM test. \(^c\) We have employed ADF test with automated lag selection, where the optimal lag length is determined using AIC. AIC selected a 5 lag model for BUX, PX and STOXX50 and a 2 lag model for WIG.

Table 2 gives the Pearson correlations (or the unconditional correlations) between index return series. The unconditional correlations among CEE markets tend to be only marginally higher than the unconditional correlations vis–à–vis euro area and reach the values about 0.6.
In terms of market capitalization, the Polish stock market is much larger than the Czech and Hungarian markets. The market capitalization of Polish stock market was approximately 110 000 million euro in 2011, while the market capitalization is approximately 30 000 and 20 000 million euro for the Czech and Hungarian stock markets, respectively. Similarly, trading volume of WSE was 5.1 times higher than the trading volume of BSE and 4.6 times higher than the trading volume of PSE in 2011. Regarding the number of initial public offerings (IPO), WSE is ranked first with 204 IPOs only in 2011, which is an activity comparable to the developed European stock markets. On the other hand, BSE and PSE typically organize about one IPO per year.

<table>
<thead>
<tr>
<th></th>
<th>BUX</th>
<th>PX</th>
<th>WIG</th>
<th>STOXX50</th>
</tr>
</thead>
<tbody>
<tr>
<td>BUX</td>
<td>1</td>
<td>0.58</td>
<td>0.61</td>
<td>0.53</td>
</tr>
<tr>
<td>PX</td>
<td>1</td>
<td>0.64</td>
<td>0.55</td>
<td></td>
</tr>
<tr>
<td>WIG</td>
<td></td>
<td>1</td>
<td>0.55</td>
<td></td>
</tr>
<tr>
<td>STOXX50</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>
4 Asymmetric DCC Model

Engle (2002) proposes the dynamic conditional correlation (DCC) model that is a direct generalization of the constant conditional correlation (CCC) model of Bollerslev (1990). The specification assumes that the $1 \times k$ vector of returns $r_t$ is conditionally normally distributed with zero mean and variance–covariance matrix $H_t$.

$$r_t|\mathcal{F}_{t-1} \sim N(0, H_t)$$

where $\mathcal{F}_{t-1}$ is the information set at time $t-1$.

The variance–covariance matrix $H_t$ can be decomposed to $H_t = D_t R_t D_t$, where $D_t$ is a diagonal matrix with the $i$-th diagonal element corresponding to the conditional standard deviation of the $i$-th asset and $R_t$ is the time–varying correlation matrix.

$$D_t = \text{diag}\{\sigma_{it}\} \quad \text{where} \quad \sigma_{it} = \sqrt{\sigma_{it}^2}$$

$$R_t = \{\rho_{ij,t}\} \quad \text{where} \quad \begin{cases} \rho_{ij,t} = 1 & \text{for} \quad i = j \\ \rho_{ij,t} \leq 1 & \text{for} \quad i \neq j \end{cases}$$

Different GARCH-type models with different lag lengths are possible for different return series. The best model is typically selected using BIC. All the GARCH specifications can be expressed in a nested form as:

$$\sigma_{it}^\delta = \omega_i + \sum_{p=1}^{P_i} \alpha_{ip}|r_{it-p}|^\beta + \sum_{a=1}^{Q_i} \gamma_{ia}|r_{it-a}|^\delta I_{[r_{it-a} < 0]} + \sum_{q=1}^{Q_i} \beta_{iq}\sigma_{it-q}^\delta$$

where $\delta = 1, 2$ depending on whether we parametrize the conditional standard deviation or the conditional variance.

The correlation dynamics is given by:

$$Q_t = \left(1 - \sum_{m=1}^{M} \theta_m - \sum_{n=1}^{N} \varphi_n\right)Q + \sum_{m=1}^{M} \theta_m (\epsilon_{t-m} \epsilon_{t-m}') + \sum_{n=1}^{N} \varphi_n Q_{t-n}$$

and

$$R_t = Q_t^{-1} Q_t^{-1}$$

where $\epsilon_t = D_t^{-1} r_t$, (or equivalently $\epsilon_t = r_t \odot \sigma_t^{-2}$) are the standardized returns. $Q = E[\epsilon_t \epsilon_t']$ is the unconditional correlation of the standardized returns and the expectations are estimated using their sample analogue $T^{-1} \sum_{t=1}^{T} \epsilon_t \epsilon_t'$. Multiplication by $Q_t' = (Q_t \odot I_k)^{-1/2}$ guarantees that $R_t$ is a well–defined correlation matrix with unitary values along the main diagonal and each off-diagonal element being less or equal to one in absolute value.

Footnotes:

3 $\odot$ denotes Hadamard division (element–by–element division).

4 $\odot$ denotes the Hadamard product (element–by–element multiplication).
The variance–covariance matrix $H_t = D_t R_t D_t$ is positive definite as long as $R_t$ is positive definite and the univariate GARCH models are correctly specified. A necessary and sufficient condition for $R_t$ to be positive definite is that $Q_t$ must be positive definite (Engle and Sheppard, 2001). The parameter restrictions, which ensure a positive definite $Q_t$ matrix, are:

1. $\sum_{m=1}^{M} \theta_m + \sum_{n=1}^{N} \varphi_n < 1$
2. $\theta_m \geq 0$ for $m = 1, 2, \ldots, M$
3. $\varphi_n \geq 0$ for $n = 1, 2, \ldots, N$

Besides the DCC model, we also consider the asymmetric DCC (ADCC) specification of Cappiello et al. (2006). The ADCC model introduces asymmetries in the correlation dynamics.

The dynamic correlation structure is given as:

$$Q_t = \left(1 - \sum_{m=1}^{M} \theta_m - \sum_{n=1}^{N} \varphi_n\right)Q - \sum_{k=1}^{K} \tau_k N + \sum_{m=1}^{M} \theta_m \left(\epsilon_{t-m} \epsilon'_{t-m}\right) + \sum_{k=1}^{K} \tau_k \left(n_{t-k} n'_{t-k}\right) + \sum_{n=1}^{N} \varphi_n Q_{t-n}$$

where $\epsilon_t$ and $Q$ are exactly as in the DCC case. $n_t = I_{[\epsilon_t < 0]} \odot \epsilon_t$, with $I_{[\epsilon_t < 0]}$ being a $1 \times k$ indicator function, which takes on the value 1 when $\epsilon_t < 0$ and 0 otherwise. In this case, unlike in the univariate processes, the asymmetric term is applicable when both indicators $I_{[\epsilon_t < 0]}$ and $I_{[\epsilon_j < 0]}$ are equal to 1 or in other words when both returns happen to be negative. $N = E[n_t n'_t]$ can be estimated using the sample analogue $N = T^{-1} \sum_{t=1}^{T} n_t n'_t$.

Positive definiteness of $Q_t$ is ensured by imposing the following restrictions:

1. $\sum_{m=1}^{M} \theta_m + \delta \sum_{k=1}^{K} \tau_k + \sum_{n=1}^{N} \varphi_n < 1$
2. $\theta_m \geq 0$ for $m = 1, 2, \ldots, M$
3. $\tau_k \geq 0$ for $k = 1, 2, \ldots, K$
4. $\varphi_n \geq 0$ for $n = 1, 2, \ldots, N$

where $\delta = \frac{1}{2} \sqrt{N} Q^{-\frac{1}{2}}$ can be estimated on sample data.

The ADCC model is estimated via maximum likelihood assuming the conditional multivariate normality. Estimation of the model is carried out using a three step procedure (see e.g. Engle and Sheppard, 2001, and Engle, 2002). In the first step, we fit $k$ univariate GARCH–type models for each return series. Then, the unconditional correlation matrix $Q$ (and the unconditional covariance matrix $N$ in case of ADCC) is estimated using the standardized returns (asymmetric standardized returns). Finally, we estimate the

$I_{[\epsilon_i < 0]}$ and $I_{[\epsilon_j < 0]}$ where $i \neq j$ are elements of $I_{[\epsilon_t < 0]}$.
parameters, which govern the correlation dynamics. Although the conditional distribution is often misspecified, quasi-maximum likelihood estimators exist, which are consistent and asymptotically normal (Engle and Sheppard, 2001).

The joint log-likelihood function is:

\[ \mathcal{L}(\theta) = -\frac{1}{2} \sum_{t=1}^{T} \left( k \log (2\pi) + \log (|H_t|) + r_t' H_t r_t \right) \]

\[ = -\frac{1}{2} \sum_{t=1}^{T} \left( k \log (2\pi) + \log (|\mathbf{D}_t \mathbf{R}_t \mathbf{D}_t'|) + r_t' \mathbf{D}^{-1}_t \mathbf{R}_t^{-1} \mathbf{D}^{-1}_t r_t \right) \]

\[ = -\frac{1}{2} \sum_{t=1}^{T} \left( k \log (2\pi) + 2 \log (|\mathbf{D}_t|) + \log (|\mathbf{R}_t|) + \epsilon_t' \mathbf{R}_t^{-1} \epsilon_t \right) \]

This function can be split into a volatility and a correlation part. For this purpose, the parameters are divided in two groups, one corresponding to the univariate GARCH parameters and the others corresponding to dynamic correlation parameters.

GARCH: \( \phi = (\phi_1, \phi_2, \ldots, \phi_k) \) where \( \phi_i = (\omega_i, \alpha_{i1}, \ldots, \alpha_{iP}, \gamma_{i1}, \ldots, \gamma_{iO}, \beta_{i1}, \ldots, \beta_{iQ}) \)

DCC: \( \psi = (\theta_1, \ldots, \theta_m, \tau_1, \ldots, \tau_k, \varphi_1, \ldots, \varphi_n) \)

In the first step \( \mathbf{R}_t \) is replaced with \( \mathbf{I}_k \), an identity matrix of dimension \( k \). Thus, the first stage quasi-likelihood becomes:

\[ Q\mathcal{L}_1 (\phi | r_t) = -\frac{1}{2} \sum_{t=1}^{T} \left( k \log (2\pi) + 2 \log (|\mathbf{D}_t|) + \log (|\mathbf{I}_k|) + r_t' \mathbf{D}^{-1}_t \mathbf{I}_k \mathbf{D}^{-1}_t r_t \right) \]

\[ = -\frac{1}{2} \sum_{t=1}^{T} \left( k \log (2\pi) + 2 \log (|\mathbf{D}_t|) + r_t' \mathbf{D}^{-2}_t \mathbf{r}_t \right) \]

\[ = -\frac{1}{2} \sum_{t=1}^{T} \left( k \log (2\pi) + \sum_{i=1}^{k} \left( \log (\sigma^2_{i,t}) + \frac{r^2_{it}}{\sigma^2_{i,t}} \right) \right) \]

\[ = -\frac{1}{2} \sum_{t=1}^{T} \sum_{i=1}^{k} \left( \log (2\pi) + \log (\sigma^2_{i,t}) + \frac{r^2_{it}}{\sigma^2_{i,t}} \right) \]

Indeed, the first stage quasi-likelihood is the sum of individual GARCH likelihoods and maximizing the joint likelihood is equivalent to maximizing each univariate GARCH likelihood individually. The second stage quasi-likelihood is estimated conditioning on first stage parameters:

\[ Q\mathcal{L}_2 (\psi | \phi, r_t) = -\frac{1}{2} \sum_{t=1}^{T} \left( k \log (2\pi) + 2 \log (|\mathbf{D}_t|) + \log (|\mathbf{R}_t|) + r_t' \mathbf{D}^{-1}_t \mathbf{R}_t^{-1} \mathbf{D}^{-1}_t r_t \right) \]

\[ = -\frac{1}{2} \sum_{t=1}^{T} \left( k \log (2\pi) + 2 \log (|\mathbf{D}_t|) + \log (|\mathbf{R}_t|) + \epsilon_t' \mathbf{R}_t^{-1} \epsilon_t \right) \]

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Given that we condition on the first stage parameters and after excluding the constant term as its first–derivative with respect to correlation parameters is zero, the second step quasi–likelihood becomes:

$$Q_{\mathcal{L}}^* \left( \psi | \hat{\phi}, r_t \right) = \frac{-1}{2} \sum_{t=1}^{T} \left( \log (|R_t|) + \epsilon'R_t \epsilon_t \right)$$

The second step parameters are retrieved by maximizing $Q_{\mathcal{L}}^*$ as:

$$\hat{\psi} = \arg\max_{\psi} Q_{\mathcal{L}}^*$$

BFGS algorithm will be used for the maximization problem.
5 Results

First, this section presents the estimates of the degree of stock market comovements. Second, we examine whether the comovements have changed during the financial crisis. Third, we analyze whether the conditional volatilities and conditional correlations move in the same direction.

We estimate four different GARCH-type models (GARCH, GJR-GARCH, AVGARCH, TGARCH) for all series and use BIC to choose between these models. The univariate models have to be properly specified in order to estimate the conditional correlations consistently (Cappiello et al., 2006). After having estimated the conditional variances, we fit the pairwise DCC models on standardized residuals $u_t = \epsilon_t \sigma_t$. This choice is made because the correlations in DCC follow a scalar BEKK–like process and it is too restrictive to apply the model on all series at once. In addition to the DCC, the ADCC model is employed. The ADCC(1,1) model is expressed as:

\[
\begin{align*}
    r_t &= \phi_0 + \phi_1 r_{t-1} + \epsilon_t \\
    \sigma_t^2 &= \omega + \alpha_1 |\epsilon_{t-1}|^\beta + \gamma_1 |\epsilon_{t-1}|^\delta I_{[\epsilon_{t-1}<0]} + \beta_1 \sigma_{t-1}^2 \\
    Q_t &= (1 - \phi_1 - \varphi_1) Q_{t-1} - \tau_1 N_t + \theta_1 (u_{t-1} u'_{t-1}) + \tau_1 n_{t-1} n'_{t-1} + \varphi Q_{t-1} \\
    R_t &= Q_t^{-1} Q_{t-1} \\
\end{align*}
\]

First, we examine the comovements among Central European stock markets. Second, we analyze the comovements between Central European stock markets and the euro area. Table 3 below presents the ADCC results (the conditional correlations equation; the mean and variance equations are available in the Appendix in Table A.1 and A.2).

<table>
<thead>
<tr>
<th>Among Central European Stock Markets</th>
<th>BUX–PX</th>
<th>BUX–WIG</th>
<th>PX–WIG</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_1$</td>
<td>0.0093**</td>
<td>0.0172**</td>
<td>0.0234***</td>
</tr>
<tr>
<td>(2.3048)</td>
<td>(2.1186)</td>
<td>(2.5582)</td>
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<tr>
<td>$\tau_1$</td>
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<td>0.0233**</td>
<td>–</td>
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<tr>
<td>(–)</td>
<td>(2.2579)</td>
<td>(–)</td>
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<tr>
<td>$\varphi_1$</td>
<td>0.9869***</td>
<td>0.9552***</td>
<td>0.9676***</td>
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<tr>
<td>(143.86)</td>
<td>(57.237)</td>
<td>(65.431)</td>
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<table>
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<th>Central European Stock Markets vis–à–vis Euro Area</th>
<th>BUX–STOXX50</th>
<th>PX–STOXX50</th>
<th>WIG–STOXX50</th>
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<tr>
<td>$\theta_1$</td>
<td>0.0371**</td>
<td>0.0222***</td>
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<td>(2.1863)</td>
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<td>–</td>
<td>–</td>
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<tr>
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<td>(–)</td>
<td>(–)</td>
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<tr>
<td>$\varphi_1$</td>
<td>0.9354***</td>
<td>0.9665***</td>
<td>0.984***</td>
</tr>
<tr>
<td>(25.672)</td>
<td>(119.869)</td>
<td>(138.922)</td>
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</tbody>
</table>

** Denotes statistical significance at 5% level and *** at 1% level. Robust t-statistics in parentheses.

$^6$ DCC is a special case of ADCC when $\tau_1 = 0$. 

---

Table 3: ADCC Estimates
In general, the asymmetries in the conditional correlations are not as widespread as in the conditional variances. The asymmetry in the conditional variances is found for all Central European stock markets (see Table A.2) because a GJR–GARCH(1,1,1) model fits the data for BUX, PX and WIG the best according to BIC. The asymmetric effect in the conditional correlation is present only for the BUX–WIG pair.

Figure 3 shows the time–varying correlations among Central European stock markets. For the BUX–PX pair, we observe the correlations between 0.3–0.5 until 2005, followed by increases in 2005–2006. In line with the results in Savva and Aslanidis (2010), the correlations remain high with the values between 0.5–0.7 after 2006. For the BUX–WIG pair, the correlations appear to be volatile until mid–2005, varying between 0.2–0.7. This is followed by a moderate increase in the value of correlations (0.4–0.8) and a reduced variation until the end of the sample. In case of PX–WIG, an increasing trend in correlations is observed for the period from mid-2003 to 2009, followed by a decrease afterwards. Overall, the results indicate that the stock market comovements have somewhat strengthened in Central Europe.

Next, Figure 4 shows the correlations of Central European stock markets vis–à–vis the euro area. The results suggest that the stock market comovements become stronger from 2001 to 2008 and, on average, remain at this level afterwards. For the WIG–STOXX50 pair, the correlations range between 0.2–0.5 prior to 2006, followed by a steady increase until 2008 when they reach a value of 0.7. Afterwards, the correlations fluctuate between
0.5–0.8. Similar trend can be observed for the BUX–STOXX50 and PX–STOXX50 pairs, too. These correlation values are very high from the international perspective. Cappiello et al. (2006) find that the conditional correlation between the U.S. and Canadian stock markets is nearly 0.8 and about 0.7 between France, Germany, the U.K. and the U.S.

Next, we examine whether the stock market correlations are higher during financial crisis vis-à-vis the pre-crisis period. For this reason, we regress the conditional correlations on a constant and a dummy variable for the crisis. The dummy takes a value of 1 from September 15, 2008 onwards, zero otherwise.

Table 4 presents our regression results. For all the pairs, the slope coefficient \( d \) is positive and statistically significant at 1% level indicating that the stock market correlation has remained at high levels during the crisis. The magnitude by which correlations are increased varies between 0.05–0.18.

Finally, we examine the relationship between conditional correlations and conditional volatilities\(^7\). If volatilities and correlations move in the same direction (i.e. the correlations are stronger when the level of risk increases), the long run risks are higher than they might appear in the short run (Cappiello et al., 2006). Following Syllignakis and Kouretas (2011), the following regression is estimated to assess this relationship:

\[
\rho_{ij,t} = \pi + \kappa_1 \sigma_{i,t} + \kappa_2 \sigma_{j,t} + \epsilon_{ij,t}
\]

\(^7\) The conditional time-varying standard deviations are available in Figure A.1.
where \( i \) corresponds to a specific Central European stock market (Czech, Polish or Hungarian market) and \( j \) to the aggregate euro area market. If \( \kappa_1 \) and \( \kappa_2 \) are positive, the correlations between Central European stock market and the euro area stock market are higher, whenever Central European and the euro area stock markets, respectively, become more turbulent. Table 5 presents the regression results. Both for the BUX–STOXX50 and PX–STOXX50 pairs we observe a positive \( \kappa_1 \) and \( \kappa_2 \), which are statistically significant at 5% level. This indicates that the correlations for these stock market pairs are stronger during high volatility periods. Whereas, for the WIG–STOXX50 pair, \( \kappa_2 \) unlike \( \kappa_1 \) is not statistically significant.
We examine these results in a greater detail by using the rolling stepwise regressions. A time window of 120 days is chosen, leading to a total of 2,413 rolling windows. The time–varying $\kappa$’s and accompanying R-squared are presented in Figures 5, 6 and 7. Most of the time $\kappa$’s are greater than zero, even though there exists time periods when they become negative. Interestingly, the R–squared varies from 0 to 0.9.

Figure 5: **Time-varying $\kappa$ coefficients for BUX–STOXX50 pair**

On the left axis are given the R–squared values, while on the right axis are given the values of time–varying parameters.

Figure 6: **Time-varying $\kappa$ coefficients for PX–STOXX50 pair**

On the left axis are given the R–squared values, while on the right axis are given the values of time–varying parameters.

---

8 See Bussière et al. (2012) for a recent application of rolling stepwise regressions to analyze the driving factors of hedge fund returns.
Figure 7: Time-varying $\kappa$ coefficients for WIG–STOXX50 pair

On the left axis are given the R-squared values, while on the right axis are given the values of time-varying parameters.
6 Concluding Remarks

In this paper, we examine the stock market comovements among three major Central European markets (the Czech Republic, Poland and Hungary) and between these markets vis-à-vis the aggregate euro area market. For this reason, we use the asymmetric DCC model by Cappiello et al. (2006). This class of multivariate GARCH models allows for asymmetric effects in the conditional variance as well as in the conditional correlation. Therefore, it can be well suited to investigate the stock market developments during the financial crisis. We complement these results by OLS regressions to assess the degree of correlations during the recent financial crises and to evaluate the relationship between conditional correlations and conditional volatilities.

Our results suggest that asymmetric volatility is common in these stock markets. Regarding the conditional correlations, we find the asymmetric effects only in the BUX–WIG pair. Therefore, asymmetries in the correlations are not as widespread as in conditional variances. Next, our results indicate that the correlations have increased over time. The increase is observed for the correlations among all Central European stock markets and also for the correlations between the Central European markets vis-à-vis the euro area. The largest increases for Central Europe are observed for the period right after these countries entered the European Union. The values of conditional correlations are very high, about 0.6–0.7 on average. The similar values are found for the correlations among developed stock markets such as between the US and Canada (Cappiello et al., 2006, Horvath and Poldauf, 2012). The correlations remain high during the financial crisis and do not fall to the values observed before the EU entry.

Finally, we investigate the relationship between the stock market correlations and volatilities, using the OLS and the rolling stepwise regression methodology. We find that the conditional correlations and conditional variances are typically positively related. This suggests that the diversification among these stock markets is disproportionally lower during turbulent times.
References


Appendix

Table A.1: AR results

<table>
<thead>
<tr>
<th></th>
<th>BUX</th>
<th>PX</th>
<th>WIG</th>
<th>STOXX50</th>
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<td>$\phi_0$</td>
<td>3.3757e-04</td>
<td>3.1314e-04</td>
<td>3.9621e-04</td>
<td>-1.8211e-04</td>
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<td>(1.0087)</td>
<td>(1.0126)</td>
<td>(1.4932)</td>
<td>(-0.64)</td>
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<tr>
<td>$\phi_1$</td>
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<td>0.0868**</td>
<td>-0.0397**</td>
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<td></td>
<td>(2.5048)</td>
<td>(4.1093)</td>
<td>(4.3834)</td>
<td>(-1.9972)</td>
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</table>

** Denotes statistical significance at 5% level. Numbers in parentheses are $t$-statistics.

Table A.2: GARCH results

<table>
<thead>
<tr>
<th></th>
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<th>WIG</th>
<th>STOXX50</th>
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<tbody>
<tr>
<td>$\omega$</td>
<td>6.7325e-06***</td>
<td>6.1022e-06***</td>
<td>2.0045e-06***</td>
<td>1.6482e-06***</td>
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<tr>
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<td>(3.605)</td>
<td>(3.703)</td>
<td>(2.901)</td>
<td>(3.104)</td>
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<tr>
<td>$\alpha_1$</td>
<td>0.055***</td>
<td>0.0643***</td>
<td>0.0402***</td>
<td>0.1046***</td>
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<tr>
<td></td>
<td>(4.257)</td>
<td>(4.659)</td>
<td>(4.778)</td>
<td>(6.404)</td>
</tr>
<tr>
<td>$\gamma_1$</td>
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<td>0.1295***</td>
<td>0.0461***</td>
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</tr>
<tr>
<td></td>
<td>(3.179)</td>
<td>(3.516)</td>
<td>(2.877)</td>
<td>(–)</td>
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<tr>
<td>$\beta_1$</td>
<td>0.8837***</td>
<td>0.8421***</td>
<td>0.9253***</td>
<td>0.8884***</td>
</tr>
<tr>
<td></td>
<td>(50.4295)</td>
<td>(40.6234)</td>
<td>(83.7986)</td>
<td>(57.096)</td>
</tr>
</tbody>
</table>

Model: GJR–GARCH GJR–GARCH GJR–GARCH GARCH

BIC: -2.7942 -2.9743 -2.9978 -3.068

*** Denotes statistical significance at 1% level. Numbers in parentheses are robust $t$-statistics.

Figure A.1: Conditional standard deviations

BUX

PX

WIG

STOXX50