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Information Disclosure in Elections with Sequential Costly Participation

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Abstract

Electoral legislation varies across countries and within countries over time, and across different types of elections in terms of how it allows publication of intermediate election results including turnout and candidates' vote shares during an election day. Using a pivotal costly voting model of elections in which voters have privately observed preferences between two candidates and act sequentially, I study how different rules for disclosing information about the actions of early voters affect the actions of later voters, and how they ultimately impact voter and candidate welfare. Comparing three rules observed in real life elections (no disclosure, turnout disclosure and vote count disclosure), I find that vote count disclosure dominates the other two rules in terms of voter welfare. I further show that each of the rules can provide a candidate with either the greatest or the least chance to win, depending on the candidate's ex-ante support.

JEL-Classification: D71, D72, D83

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1 Introduction

Should authorities allow disclosure of more or less information to voters about the actions of early voters during an election day? Electoral legislation varies considerably, both across and within countries over time and in different types of elections in terms of what is allowed to become public information during an election day, such as turnout and candidate vote shares, or data that can characterize the outcome of a race, such as exit polls. In today's national-level elections, the majority of countries, including Germany, France, Italy, India and Russia, do not allow announcements of actual intermediate results (distribution of votes across candidates) or publication of exit poll results, but instead, every few hours they allow announcements of cumulative turnout. In some countries, such as the United States, Finland, Denmark, and Sweden, both cumulative turnout and the results of exit polls can be announced during an election day.[1] On the contrary, in countries like China and Israel, disclosure of any relevant information before the polls close is illegal.

These differences in electoral legislation persist, despite a long history of widespread political debate on information disclosure during elections. Although the primary focus of most debates is exit polls and publication of their results[2], polling can be considered a part of a broader discussion of what information about the actions of early voters should ideally be disclosed to later voters. Contemporary electoral technologies and practices provide electoral officials with a variety of tools for informing voters when needed. For example, with electronic voting technologies, which have been widely adopted in a number of countries including Finland, Norway, Estonia and others, voters can easily be provided with actual live results of elections throughout an election day.

[1] Since the 1980s, major US news organizations have agreednot to release any exit poll data before all of the polls in a state have closed. Nevertheless, "early leaks" are legal, were widespread in the past, and still occur today. See, for example, https://en.wikipedia.org/wiki/Exit_poll#Criticism_and_controversy (retrieved on 15.01.2020) for further examples, details, and reference.

[2] See, for example, "Comparative Study of Laws and Regulations Restricting the Publication of Electoral Opinion Polls" (https://www.article19.org/data/files/pdfs/publications/opinion-polls-paper.pdf, retrieved on 15.01.2020) and more recent "The Freedom to Publish Opinion Poll Results" (https://wapor.org/wp-content/uploads/WAPOR_FTP_2012.pdf, retrieved on 15.01.2020) for reviews of exit poll legislation in different countries and controversial cases of election day publication of the results of exit polls.

Although it is clear that any information on the actions of early voters is likely to affect the actions of later voters, research that would explicitly highlight mechanisms behind this relationship and, thus, its impact on election outcomes and welfare, has been limited. In this paper, using a pivotal costly voting framework, motivated by real life policies relating to information disclosure, I study how different information regimes in elections with sequential participation affect voters' decisions to cast votes, and what impact they have on candidate and voter welfare.

A large class of voting literature focuses on comparisons of simultaneous and sequential voting mechanisms, which can be thought of as voting under no information disclosure and voting under full information disclosure, respectively, in terms of how the mechanisms aggregate voters' private knowledge about characteristics of the candidates. Starting from Dekel and Piccione (2000), who showed that when voting between two alternatives is costless, the sets of equilibria in simultaneous and sequential games are identical, there has been a large body of common value models[3] designed to compare these two mechanisms. Battaglini (2005) demonstrates that when voters can abstain, an arbitrarily small cost of voting leads toa unique symmetric equilibrium under the simultaneous voting mechanism, which dominates all equilibria under the sequential mechanism in terms of the quality of information aggregation. Battaglini, Morton, and Palfrey (2007) explore, both theoretically and experimentally, the tradeoff between information aggregation, efficiency, and equity in simultaneous and sequential voting. They find that sequential voting aggregates information better, and is generally more efficient, but at the expense of early voters who bear higher participation costs. Callander (2007) and Hummel and Knight (2015) find that, in elections with a strong favorite, a sequential voting mechanism can aggregate information better than a simultaneous one, while in ex-ante tight elections, simultaneous voting is preferred.

In contrast to these and some other studies which use common value models, Bognar, Borgers and Meyer-ter-Vehn (2015) study the relative efficiency of simultaneous and sequential voting in another popular framework (first used in Palfrey and Rosenthal 1983,

[3] In common value models, voters share the same preferences but may have different beliefs about which candidate is the best in terms of unobservable relevant features such as competence. If all features of the candidates were known to everyone, voters would agree on a specific candidate.

1985; and then further developed by Borgers, 2004; and Krasa and Polborn, 2009) in which voters value the candidates differently, have heterogeneous privately observed participation costs and decide only whether to abstain or to vote for their preferred candidate. In models of this type, although voters have different preferences for candidates, the exact distribution of support for the candidates is unknown. Instead, a voter may support a candidate with a commonly known probability, which thus is a measure of the candidate's ex-ante support. Bognar et al. (2015) look for a voting mechanism which maximizes expected voter welfare and find that sequentially cast ballots are one feature of an optimal mechanism[4] when candidate support is equal ex-ante. Although this result favors sequential voting over simultaneous voting, the authors admit that it cannot be generalized to situations in which the support is not equal ex-ante.

In this paper, I also use a private value costly voting model with two candidates, but I allow for arbitrary ex-ante support. I make the ex-ante support the central component of the model, exploring how its values affect the outcomes. Furthermore, being motivated by real-life elections and policies related to information disclosure in elections, I study three distinct information regimes: full information disclosure (voters acting later observe all actions of early voters), partial disclosure (later voters observe only the early voter turnout), and no information disclosure. While voting under the no disclosure regime in my model is equivalent to simultaneous voting, the full disclosure regime differs from pure sequential voting. Instead of allowing every voter to sequentially decide whether to cast a ballot or to abstain, observing the actions of all previous voters, I divide the voters into two groups. Within each group, voters vote simultaneously, while voters in the later group may observe some actions of the early group of voters, depending on the disclosure regime applied. This assumption simplifies the analysis considerably, though it is not much less realistic than pure sequential voting. A similar assumption is used by Battaglini (2005), who allows a fixed number of voters to simultaneously vote in each period in his sequential voting model.

[4] The other features are: arbitrarily chosen default decision, interpretation of abstention as a vote for the default option, and termination of voting when one candidate has received a certain number of votes.

Another important assumption of my model shared with all the models of sequential voting mentioned above is exogenous assignment of voters to their places in the voting sequence. While real-life voters are generally free to choose at what point of an election day to cast their votes, there are often natural restrictions that limit their choices, such as working hours or time zones.[5] To my knowledge, Dekel and Piccione (2014) is the only study that allows voters to choose the time at which they cast their votes. In their model, voters have different preferences for multiple candidates, do not have participation costs, and decide only whether to cast a vote early or later. They found that sequential voting may arise endogenously only when voters vote strategically, not for their first best candidate. Otherwise, all voters prefer to postpone casting their votes and thus voting becomes simultaneous. In this paper, I consider voters' assignment to sequential voting groups as a given, in order to focus on the effect of information available to voters on their participation decisions and, thus, on welfare.

I compare equilibria under three distinct information regimes: full information (votes) disclosure, partial (turnout) disclosure, and no information disclosure. I present an analytical solution for the case with two sequentially acting voters with uniformly distributed costs, and then describe a general model with N voters, K of whom act early. Next, I present general analytical results, and solve the model numerically for various values of N and K to highlight a number of consistent findings. I compare two characteristics of equilibria under each regime as functions of candidates' ex-ante support – expected average voter welfare and candidates' probability of winning elections.

First, I find that the full disclosure regime dominates the other two regimes in terms of voter welfare. Second, partial and no disclosure regimes become almost equivalent in terms of both voter and candidate welfare even for a relatively small number of voters, except for the extreme values of candidates' ex-ante support. Third, when ex-ante support for one of the candidates approaches 1, the effects of partial disclosure converge to those of full disclosure. Finally, I demonstrate that, in terms of the probability of winning, full disclosure benefits the candidate ex-ante preferred by the majority and hurts the ex-ante minority candidate; the minority

[5] For example, in Russia the time difference between the most western (Kaliningrad) and the most eastern regions (Kamchatka) is 10 hours. Given that the polling stations in Russian elections are typically open from 8 A.M. till 8 P.M., when polling stations in the West open, voting is almost over in the East.

candidate benefits the most from partial disclosure when his/her support is relatively high, and from no disclosure when his/her ex-ante support is relatively low. Hence, the choice of information regime made by a candidate who can set the agenda may signal his/her expectations regarding his/her voter support.

With this paper, I contribute to the voting literature is several ways. First, I explicitly study real life policies with information disclosure, including “turnout only disclosure” which has not been analysed previously. Second, I focus not only on voter welfare, which is typical for analyses of electoral institutions, but I also study what impact different information regimes may have on election outcomes and hence on candidate welfare. Finally, I perform an analysis for the whole range of potential values of candidate support among voters, which allows me to illustrate that the impact of information regimes varies in a non-straightforward way.

2 General Setup

I analyze participation in elections under different information regimes in a pivotal costly voting framework. Elections are modeled similarly to those studied in a large body of literature, in which voters are assumed to make participation decisions based on the probability that their votes will alter election outcomes. Costly private value voting models of a similar type have been pioneered by Palfrey and Rosenthal (1983, 1985) and Ledyard (1984), and widely used to study voter turnout as well as electoral policies and institutions by, for example, Borgers (2004), Krasa and Polborn (2009), Ghosal and Lockwood (2009), Taylor and Yildirim (2010a,b), Aguiar-Conraria and Magalhaes (2010) and more recently by Bognar et al. (2015), Kartal (2015), Krishna and Morgan (2015), Vorobyev (2016), Arzumanyan and Polborn (2017), Grillo (2017), and Chakravartya, Kaplana and Myles(2018).

There are two candidates, A and B, and $N \geq 2$ voters. Voters have preferences for candidates but the exact distribution of their support is unknown. Instead, a voter may support candidate A with commonly known probability $\alpha \in [0,1]$ or candidate B with probability $1 - \alpha$. Parameter α is the key element of the paper: I study how equilibria and their characteristics change in response to changes in α which is, in fact, a measure of the candidates' ex-ante support.

Each voter votes in either period $t = 1$ or $t = 2$, that is, either early or later. Assignment of voters to voting periods is exogenous: there are $K < N$ voters voting early and $N - K$ voting later. Every voter knows the period in which he or she votes, and K is common knowledge. I refer to a voter as "T_t-type" if his/her preferred candidate is $T \in \{A, B\}$, and he or she acts in period $t \in \{1,2\}$. Each voter i has an individual specific voting cost c_i drawn from a commonly known distribution F over interval $[c_{min}, c_{max}]$, independently of his/her type, voting period and the other voters. The distribution function F is assumed to be continuous and to have full support over $[c_{min}, c_{max}]$. If a voter's preferred candidate wins, the voter gains utility 1 if he or she did not vote, and $1 - c_i$ otherwise. If his/her favored candidate loses, the voter gains utility 0 if he or she abstained, and $-c_i$ if he or she voted. Elections are run under majority rule and a tie is resolved with a coin flip.

The analysis of voter behavior should begin from the observation that, conditional on voting, a voter's weakly dominant strategy is to vote for his/her preferred candidate; thus, I focus on participation decisions only. A voter decides to vote if and only if his/her participation cost does not exceed his/her expected benefit from participation. The benefit depends on the probability that his/her vote will be pivotal (decisive). Since, conditional on being pivotal, a voter can either turn a loss into a tie or a tie into victory, gaining the additional utility of 0.5 in each case, he or she votes if and only if $0.5\Pi \geq c_i$ where Π is the probability of being pivotal and c_i is his/her participation cost. Throughout the paper, I focus on within-group symmetric equilibria where all voters of the same type and with the same information adopt the same voting strategy, casting the vote if the participation cost appears to be below a certain threshold and abstaining otherwise.[6] Hence, for every voter, Π is a function of a set of strategies, $c_A^1, c_A^2(s), c_B^1, c_B^2(s)$, where $c_T^t(s)$ is a cost threshold such that a voter who supports candidate T, acts in period t and has information s, votes if his/her voting cost is $c_i \leq c_T^t$, and abstains otherwise.

Given participation thresholds, the ex-ante probability that a voter of type T_t votes is $p_T^t = F(c_T^t)$, $T \in \{A, B\}$, $t \in \{1,2\}$. Since these are the participation probabilities, not the cost thresholds themselves, which is fundamental to the questions studied in this paper, I perform a further analysis of the participation probabilities p_T^t. If needed, the cost thresholds can always be recovered from the probabilities as $c_T^t = F^{-1}(p_T^t)$.

I calculate and compare equilibria under three distinct regimes of information disclosure. Under no information (N-regime), later voters know nothing about what early voters did, which effectively makes voting simultaneous. Under partial information (P-regime), later voters only know how many early voters chose to vote, but do not know who they voted for. Under full information (F-regime), later voters know both the turnout of early voters and the distribution of their votes across candidates.

[6] An alternative approach sometimes used in pivotal costly voting literature (see, for example, Taylor and Yildirim, 2010b or Arzumanyan and Polborn, 2017) is to assume that the voting cost is the same for every voter and to allow for a mixed strategy equilibrium, in whichvoters mix between voting and abstaining with type-specific probabilities. While being marginally less intuitive, such an approach reduces the dimensionality of the problem and allows the researcher to avoid making any assumption on the distribution of costs. However, in the case of this paper, the reduction in the difficulty of the problem would not be sufficient to allow for any additional result. Therefore, I assume heterogenous voting costs, to be consistent with the majority of the pivotal costly voting literature.

3 Two-Voter Model

First, consider a simplified version of the model described, in which there are two voters acting sequentially, whose voting costs are independently drawn from a uniform distribution over $[0,0.5]$, implying that $F(c) = 2c$ and $F^{-1}(p) = 0.5p$.

3.1 Equilibrium

Under **no information** disclosure, the game is equivalent to simultaneous voting, and the equilibrium strategy of each voter depends only on the candidate he or she supports, and not on the time period in which he or she votes: $p_A^1 = p_A^2 = p_A$ and $p_B^1 = p_B^2 = p_B$. A voter is always pivotal except in the case in which another voter supports the same candidate and votes. The equilibrium values of the participation probabilities thus must solve the following system:

$$0.5(1 - \alpha p_A) \geq 0.5p_A; 0.5(1 - (1 - \alpha)p_B) \geq 0.5p_B, \quad (1)$$

with equality when $c_T = 0.5p_T < c_{max} = 0.5$.

The solution of this system is

$$p_A^1 = p_A^2 = \frac{1}{1 + \alpha}; p_B^1 = p_B^2 = \frac{1}{2 - \alpha}.[7] \quad (2)$$

Under **partial information** disclosure, the action of the second period voter depends on the information he or she has after the first period. If he or she observes 0 turnout, he or she is always pivotal, expects to gain utility 0.5 from voting, and thus always votes, since his/her participation cost cannot exceed 0.5, i.e. $p_A^2(0) = p_B^2(0) = 1$. If the observed turnout is 1, the equilibrium strategies of the second period voter must solve:

$$0.5(1 - \gamma) \geq 0.5p_A^2(1); 0.5\gamma \geq 0.5p_B^2(1), \quad (3)$$

with equality when $c_T^2(1) = 0.5p_T^2(1) < c_{max} = 0.5$. In the above formula, γ is the posterior probability that candidate A will lead after the first period, or equivalently, that the first period

[7] Note that candidate A and candidate B differ only in the ex-ante probabilities that a voter is a supporter. Since these probabilities must add up to 1, any formula relevant for candidate B or his/her supporters can be obtained by substituting α with $1 - \alpha$ in the corresponding formula for candidate A. For example, the above expression for equilibrium participation p_B^1 can be derived by such a substitution from the expression for p_A^1.

voter is an A-supporter, conditional on the observed turnout being 1: $\gamma = \frac{\alpha p_A^1}{\alpha p_A^1 + (1-\alpha) p_B^1}$. Since it can take values within the $[0,1]$ interval only, inequalities (2) are always satisfied with equality guaranteeing an internal solution.

Further, the first period voter should anticipate that his/her participation will change the information available to the second period voter. Since, if the first period voter abstains, the second period voter always votes regardless of his/her type, there are only two cases in which the first period A-supporter might be pivotal, and in both cases the second period voter must be a B-supporter. First, the second period B-supporter votes after observing that the first period voter voted. In this case, the A-supporter's participation would turn the loss of candidate A into a tie, giving an additional expected utility of 0.5. Second, if the second period B-supporter abstains after observing that the first period voter participated, the A-supporter's participation would turn a loss into victory, providing him/her an additional utility of 1. Therefore, the first period A-supporter's equilibrium participation is given by the following condition:

$$(1-\alpha)(0.5 p_B^2(1) + 1 - p_B^2(1)) \geq 0.5 p_A^1. \quad (4)$$

Similarly, for the first period B-supporter:

$$\alpha(0.5 p_A^2(1) + 1 - p_A^2(1)) \geq 0.5 p_B^1. \quad (5)$$

Solving the system of inequalities (3)–(5), one can obtain the unique equilibrium:

$$p_A^1 = \min\{(1-\alpha)(2-\gamma), 1\}; \quad p_B^1 = \min\{\alpha(1+\gamma), 1\};[8] \quad (6)$$

$$p_A^2(0) = p_B^2(0) = 1; \quad p_A^2(1) = 1-\gamma; \quad p_B^2(1) = \gamma; \quad (7)$$

where

$$\gamma = \begin{cases} 0, \alpha = 0, \\ \frac{\alpha - 2 + \sqrt{\alpha^2 + 8(1-\alpha)}}{2(1-\alpha)}, 0 < \alpha < \frac{1}{3}, \\ 0.5, \alpha \in [\frac{1}{3}; \frac{2}{3}], \\ 1 - \frac{(1-\alpha) - 2 + \sqrt{(1-\alpha)^2 + 8\alpha}}{2\alpha}, \frac{2}{3} < \alpha < 1, \\ 1, \alpha = 1, \end{cases} \quad (8)$$

[8] Note that, for $\alpha \in [\frac{1}{3}, \frac{2}{3}]$, $p_A^1 = \frac{3}{2}(1-\alpha)$ and $p_B^1 = \frac{3}{2}\alpha$, and both values are within [0,1] interval. While for $\alpha < \frac{1}{3}$ and $\alpha > \frac{2}{3}$, one of the two participation probabilities is bound by 1: $p_A^1 = 1$ in the former case, and $p_B^1 = 1$ in the latter case.

and γ is the posterior probability that candidate A will lead after the first period, conditional on the observed turnout being 1.[9]

Under **full information** disclosure, the second period voter always abstains when his/her preferred candidate leads after the first period and always participates otherwise. Anticipating this response, the first period voter is pivotal whenever the second period voter supports the opposing candidate. Hence, the equilibrium strategies under F-regime are:

$$p_A^1 = 1 - \alpha; p_B^1 = \alpha; p_A^2(+) = p_B^2(-) = 0; p_A^2(0) = p_A^2(-) = p_B^2(0) = p_B^2(+) = 1; \quad (9)$$

where $+(-)$ denotes the situation, when candidate A (B) leads after the first period and (0) stands for a tie.

Given voter participation probabilities, one can analyze equilibrium candidate and voter welfare under different information regimes and thus compare the regimes from the welfare perspective.

3.2 Candidate Welfare

Since the candidates care only about winning the election, their welfare functions are a strictly increasing function of the corresponding winning probabilities. The winning probability of a candidate is the probability that he or she will receive more votes than another candidate plus the probability of a tie multiplied by 0.5. In the two-voter case, under any information regime, candidate A wins when 1) both voters are A-supporters except when neither of them votes and the coin flip favors B; 2) both voters are B-supporters, neither of them votes, and the coin flip favors A; 3) one voter supports A, one voter supports B, and only the A-supporter votes; 4) one voter supports A, one voter supports B, either both or neither of them votes, and the coin flip favors A. Given equilibrium voter strategies

[9] Since this is a game of incomplete information, I use the concept of perfect Bayesian equilibrium for the solution. The concept requires specification of what the second period voter should think about the first period score after observing turnout, which is a non-trivial task only when the turnout is 1. Given the strategies of the first period voter, the second period voter, regardless of his/her preferred candidate, should think that candidate A leads 1:0 with probability γ, which is derived using Bayes' rule. When $\alpha = 0$ $(\alpha = 1)$, i.e. when both voters support B (A) with certainty, the first period voter should never vote. Therefore, observing turnout 1 after the first round is off the equilibrium path, and hence the concept of perfect Bayesian equilibrium requires the beliefs to be arbitrary in this case. Nevertheless, the only reasonable belief is $\gamma = 0$ $(\gamma = 1)$, since if the first period voter votes, he or she can only be a B (A) supporter.

for each information regime, one may obtain the following expressions (see the Appendix for the derivation) for candidate A's winning probabilities:

$$W_A^N = \frac{1}{2} + \frac{3}{2}\frac{2\alpha - 1}{(2-\alpha)^2(1+\alpha)^2}. \tag{10}$$

$$W_A^P = \alpha + \frac{1}{2}\alpha(1-\alpha)((2-\gamma)p_A^1 - (1+\gamma)p_B^1), \tag{11}$$

where γ is given by formula (8), and p_A^1 and p_B^1 are the equilibrium strategies (6).

$$W_A^F = \frac{1}{2}\alpha\left(1 + \alpha + \frac{(1-\alpha)^2}{2}\right). \tag{12}$$

The winning probabilities for candidate B for each regime can be obtained either as $1 - W_A$ or by substituting α with $1 - \alpha$ in the expressions for W_A.

Figure 1 illustrates A's winning probability as a function of α under each regime.

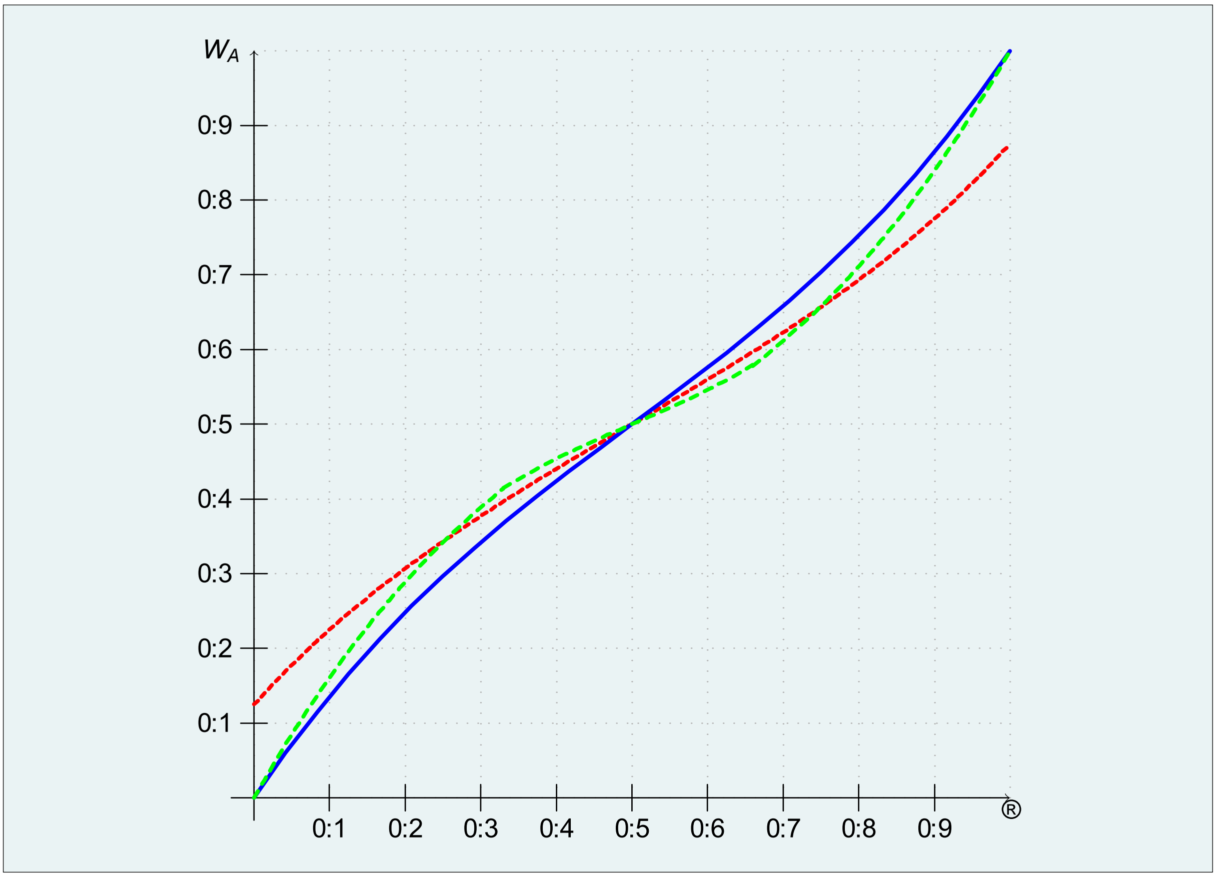


Figure 1: A's winning probability: No information (dotted), partial information (dashed), full information (solid)

One may see that, whenever A's ex-ante support is above 0.5, the candidate is better off under a full disclosure regime, while when $\alpha < 0.5$, full disclosure is his/her worst alternative. This result is intuitive: when a candidate has strong ex-ante support (high values of α), he or she wants the second period voter to know what happened in the first period, since in the case of bad luck (the first period voter abstained or appeared to be a supporter of the rival candidate and voted) the second period voter who is likely to be the candidate's supporter can step in.[10]

However, when the ex-ante support is less than 0.5, the candidate does not always benefit from as little information disclosure as possible: for the values of α above a certain threshold, $\alpha^* < \alpha < 0.5$, partial information disclosure is the candidate's most preferred regime. Likewise, when A has ex-ante majority support, no disclosure is not his/her worst alternative as long as α is relatively small, i.e. $0.5 < \alpha < 1 - \alpha^*$.

To see why, for example, no disclosure is not always the worst alternative for a candidate with $\alpha > 0.5$, consider what happens when α increases from the value of 0.5. At $\alpha = 0.5$, the model is fully symmetric and hence both candidates are ex-ante equally likely to win the election regardless of the information regime. When α increases, two things happen simultaneously. First, every voter becomes more likely to support candidate A which, other things being equal, gives A more votes in expectation. I refer to this as the "support" effect. Second, equilibrium participation probabilities also change. I refer to this as the "participation" effect. Specifically, an increase in α results in greater participation by B-supporters and less participation by A-supporters, implying a lower probability that A will win. Indeed, for each regime the "support" effect is the first order effect, and it is always stronger than the "participation" effect, guaranteeing that a candidate always receives net benefits from having ex-ante more supporters, that is W_A is strictly increasing in α for all regimes. It can be shown (see the Appendix for details) that while, given equilibrium participation thresholds, the positive "support" effect from an increase in α from 0.5 on A's winning probability is larger for the P-regime than for the N-regime (1 against 8/9, due to greater expected participation under the P-regime than under the N-regime), the negative "participation" effect is much

[10] One may think of an extreme case when $\alpha = 1$, where candidate A wins with certainty under full disclosure since the first period voter always abstains and the second period voter, who is necessarily an A-supporter, always votes. However, under no disclosure, both voters will vote with probability of less than 1, and hence there is a chance that both of them will abstain and A will lose the coin flip.

stronger for the P-regime (–9/16 against –8/27). As a result of this significant difference in the "participation" effects, candidate A is better off under the N-regime than under the P-regime for α just above 0.5. With higher α, while the differences in both "support" and "participation" effects become even larger, at some point, when $\alpha > 1 - \alpha^*$, the difference in the "support" effects outweighs the difference in the "participation" effects, making candidate A more likely to win under a P-regime than under an N-regime.

Finally, note that A's winning probabilities under the P regime converge to A's winning probability under the F regime when α approaches 0 or 1, since in these cases, observing turn out for the second-round voter is as informative as observing actual votes. This is because when the second period voters know that $\alpha = 1$, that is, the first period voter can only be an A-supporter, observing turnout equal to 1 implies that A leads 1:0, while when $\alpha = 0$, the same turnout means that A loses 0:1.

3.3 Voter Welfare

When voting is costly, participation implies a tradeoff between the quality of the aggregation of voters' preferences and participation costs. Higher participation increases the probability that the candidate preferred by the majority will be elected, but at the same time it implies larger total costs borne by voters. To compare the three regimes of information disclosure from the perspective of voter welfare, note that the expected utility of a voter who supports candidate $T \in \{A, B\}$, acts in period $t \in \{1,2\}$, and has information s can be expressed as follows:

$$EW_T^t(s) = \int_0^{c_T^t(s)} (u_T^t(s) - c)\, \mathrm{d}F(c) + \int_{c_T^t(s)}^{c_{max}} v_T^t(s)\, \mathrm{d}F(c) = v_T^t(s) + \int_0^{c_T^t(s)} (u_T^t(s) - v_T^t(s) - c)\, \mathrm{d}F(c), \qquad (13)$$

where $u_T^t(s)$ is the voter's expected benefit when he or she participates, $v_T^t(s)$ is his/her expected benefit when he or she abstains, and $c_T^t(s)$ is his/her equilibrium participation threshold. In terms of participation probabilities p_T^t, the formula above can be expressed as:

$$EW_T^t(s) = v_T^t(s) + p_T^t(s)(u_T^t(s) - v_T^t(s)) - \frac{1}{4}(p_T^t(s))^2. \qquad (14)$$

Then, average ex-ante voter welfare is:

$$EW = \frac{1}{2}\left(\alpha EW_A^1 + (1-\alpha)EW_B^1 + \alpha \sum_s \gamma_s \, EW_A^2(s) + (1-\alpha)\sum_s \gamma_s \, EW_B^2(s)\right), \quad (15)$$

where γ_s is the ex-ante probability of getting information s in the second period: $s \in \{0,1\}$ under the P-regime, $s \in \{0,-,+\}$ under the F-regime, and $EW_T^2(s) = EW_T^1$, $T \in \{A,B\}$ under the N-regime.

It can be shown (see the Appendix for the derivations) that:

$$EW^N = \frac{1}{4}\left(1 + \frac{2-\alpha^2(2-\alpha)}{(1+\alpha)^2} + \frac{2-(1-\alpha)^2(1+\alpha)}{(2-\alpha)^2}\right). \quad (16)$$

$$EW^P = \begin{cases} \frac{1}{8}[3 + 4(1-\alpha)^2 + \alpha(3 - 2(1-\alpha)\gamma + \alpha(1-\alpha)(1+\gamma)^2) + \\ +(1-\alpha)(1+\alpha(2-\gamma))(\alpha(1+\gamma)^2 + (1-\alpha)(2-\gamma)^2 - 3)], \alpha < \frac{1}{3}, \\ \frac{7}{8} - \alpha(1-\alpha), \alpha \in [\frac{1}{3};\frac{2}{3}], \\ \frac{1}{8}[3 + 4\alpha^2 + (1-\alpha)(3 - 2\alpha(1-\gamma) + \alpha(1-\alpha)(2-\gamma)^2) + \\ +(1-\alpha)(1+\alpha(2-\gamma))(\alpha(1+\gamma)^2 + (1-\alpha)(2-\gamma)^2 - 3)], \alpha > \frac{2}{3}, \end{cases} \quad (17)$$

where γ is given by formula (8).

$$EW^F = \frac{7}{8} - \alpha(1-\alpha). \quad (18)$$

Figure 2 illustrates voter welfare as a function of α under each information regime. One may see, both from the figure and the formulas (9) and (10) that no information disclosure is always dominated by the other two regimes, and for $\alpha \in [1/3,2/3]$ welfare under partial information and welfare under full information are equivalent. For $\alpha < 1/3$ and $\alpha > 2/3$, however, partial disclosure delivers even higher welfare than full disclosure. This is a result of the corner solution that arises in these cases: when, for example, α decreases, participation by the first period A-supporter increases and reaches its maximum at $\alpha = 1/3$; when α decreases further, participation cannot continue to increase. This indicates that, under partial disclosure, at least the participation of the first period voter is inefficiently high since his/her inability to increase it generates a welfare gain.[11] In the General Model section, I show that with larger numbers of voters, corner solutions disappear, and welfare under partial disclosure converges to welfare under no disclosure almost everywhere except under the extreme values of α, leaving full disclosure as the best information regime from the perspective of voter welfare.

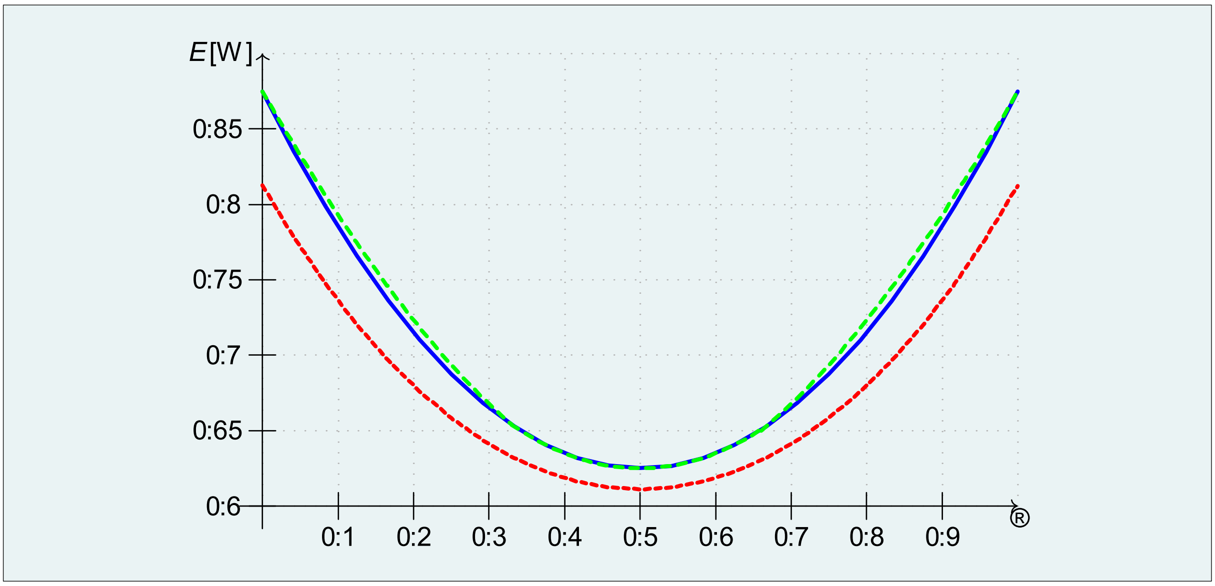


Figure 2: Average expected voter welfare: No information (dotted), partial information (dashed), full information (solid)

[11] Although efficient participation under costly voting is aninteresting and relatively well studied topic (see, for example, Borgers 2004, and Krasa and Polborn 2009), in this paper I focus only on the efficiency of information regimes.

4 General Model

4.1 Setup and Equilibrium

Consider a more general model in which the total number of voters is $N > 2$, $K < N$ of them may vote early, and costs are drawn from a distribution F over $[c_{min}, c_{max}]$, $0 \leq c_{min} < c_{max} < 1$. I construct the voters' pivotal probabilities for each information regime.

No disclosure regime

Suppose that all A-supporters adopt the voting strategy c_A, i.e. an A-voter votes if his/her voting cost is below c_A and abstains otherwise. Similarly, suppose B-voters adopt strategy c_B. Then, the probability that a randomly chosen voter will vote is $p_A = F(c_A)$ and $p_B = F(c_B)$ for A-types and B-types respectively.

Denote $P_i^j(k) = \binom{j}{i} k^i (1-k)^j$ for shorter notation. Consider an A-type voter. Then, the probability that there are a A-types among other $N-1$ voters is $P_a^{N-1}(\alpha)$. The probability that l of them will participate in elections is $P_l^a(p_A)$. An A-supporter is pivotal whenever the number of B-participants is equal to or exceeds the number of A-participants by 1. The probability that an A-supporter will be pivotal is then:

$$\Pi_A(p_A, p_B) = \sum_{a=0}^{N-1} \sum_{l=0}^{a} P_a^{N-1}(\alpha) P_l^a(p_A) \left(P_l^{N-a-1}(p_B) + P_{l+1}^{N-a-1}(p_B) \right). \tag{19}$$

Similarly, one can construct a pivotal probability function for a B-supporter:

$$\Pi_B(p_A, p_B) = \sum_{a=0}^{N-1} \sum_{l=0}^{a} P_a^{N-1}(\alpha) P_l^a(p_A) \left(P_l^{N-a-1}(p_B) + P_{l-1}^{N-a-1}(p_B) \right).$$

Since whenever a voter is pivotal his/her participation increases his/her utility by 0.5 (by voting he or she either turns a tie into a win or a loss into a tie), equilibrium values of p_A and p_B are the solution for the following system:

$$0.5\Pi_A(p_A, p_B) \geq F^{-1}(p_A), 0.5\Pi_B(p_A, p_B) \geq F^{-1}(p_B), \tag{20}$$

with equalities when $F^{-1}(p_A) = c_A < c_{max}$ and $F^{-1}(p_B) = c_B < c_{max}$ respectively.

Partial disclosure regime

Under a partial disclosure regime, voters from the group acting later observe the turnout among voters acting early, and condition their actions on the observed number of the first period votes cast. Denote this number as $d \in \{0, .., K\}$ and consider a first period A-supporter. The voter takes the participation probabilities of other first period voters as given, and anticipates that the actions of second period voters depend on the observed number of first period participants, which he or she is affecting. Since his/her participation changes the participation of all second period voters, he or she also anticipates that there are 6 potential situations in which he or she is pivotal. By participating, a first period A-supporter can turn the loss of candidate A in the case of the voter's abstention into either a tie (increasing his/her expected utility by 0.5) or into a victory (increasing by 1); a tie into a victory (increasing by 0.5) or into a loss (decreasing by 0.5); and a victory into a tie (decreasing by 0.5) or a loss (decreasing by 1). Then, calculating the probabilities of each potential pivotal situation, his/her expected benefit from voting is:

$$B_A^1 = \sum_{a_1=0}^{K-1} \sum_{l=0}^{a_1} \sum_{m=0}^{K-a_1-1} \sum_{a_2=0}^{N-K} \sum_{i=0}^{a_2} \sum_{v=0}^{a_2} P_{a_1}^{K-1}(\alpha) P_l^{a_1}(p_A^1) P_m^{K-a_1-1}(p_B^1) P_{a_2}^{N-K}(\alpha) \cdot \tag{21}$$
$$\cdot P_i^{a_2}(p_A^2(l+m)) P_v^{a_2}(p_A^2(l+m+1))(X+Y+Z)),$$

where, denoting $b_2 = N - K - a_2$,

$$X = \sum_{j=l-m+i+1}^{b_2} P_j^{b_2}(p_B^2(l+m)) \left(\frac{1}{2} P_{l-m+v+1}^{b_2}(p_B^2(l+m+1)) + \sum_{w=0}^{l-m+v} P_w^{b_2}(p_B^2(l+m+1)) \right), \tag{22}$$

the utility from the cases in which the voter's participation turns A's loss into a tie or a victory;

$$Y = \frac{1}{2} P_{l-m+i}^{b_2}(p_B^2(l+m)) \left(\sum_{w=0}^{l-m+v} P_w^{b_2}(p_B^2(l+m+1)) - \sum_{w=l-m+v+2}^{b_2} P_w^{b_2}(p_B^2(l+m+1)) \right), \tag{23}$$

the utility from the cases in which the voter's participation turns a tie into A's victory or loss;

$$Z = -\sum_{j=0}^{l-m+i-1} P_j^{b_2}(p_B^2(l+m)) \left(\frac{1}{2} P_{l-m+v+1}^{b_2}(p_B^2(l+m+1)) + \sum_{w=l-m+v+2}^{b_2} P_w^{b_2}(p_B^2(l+m+1)) \right), \tag{24}$$

the utility from the cases in which the voter's participation turns A's victory into a tie or a loss. Likewise, one can construct expected benefits for a first period B-supporter.

Consider a second period A-supporter who observes turnout $d \in \{0,..,K\}$. his/her expected benefit from voting is then:

$$B_A^2(d) = \frac{1}{2}\sum_{i=-d}^{d}\sum_{a_2=0}^{N-K-1}\sum_{l=0}^{a_2}\omega\,(i|d)P_{a_2}^{N-K-1}(\alpha)P_l^{a_2}(p_A^2(d))\big(P_{l+i}^{b_2-1}(p_B^2(d)) + P_{l+i+1}^{b_2-1}(p_B^2(d)ight)), \tag{25}$$

where $\omega(i|d)$ is the posterior probability that the vote difference between A and B after the first round will be i, given that d votes were cast, calculated using Bayes' rule.[12]

The equilibrium is then given by the following system of 2(K+1)+2 inequalities:

$$B_A^1 \geq c_A^1, B_B^1 \geq F^{-1}(p_B^1), \tag{26}$$

$$B_A^2(0) \geq F^{-1}(p_A^2(0)), \dots, B_A^2(K) \geq F^{-1}(p_A^2(K)),$$

$$B_B^2(0) \geq F^{-1}(p_B^2(0)), \dots, B_B^2(K) \geq F^{-1}(p_B^2(K)),$$

with equalities when $F^{-1}(p_T^t(d)) = c_T^t(d) < c_{max}$, $t \in \{1,2\}$, $T \in \{A,B\}$, $d \in \{0,..,K\}$.

Full disclosure regime

Under a full disclosure regime, voters from the group acting second observe all the actions of the early voters. However, the only thing that matters for their decisions is the difference in the number of votes cast for each candidate. Denote this integer difference between votes for A and votes for B in the first period as $d \in \{-K,..,K\}$.

Consider an A-supporter who acts early. Taking as a given the participation probabilities of A-supporters and B-supporters acting early and anticipating that voters acting later will condition their actions on the observed d, which is affected by the voter's action, the expected voter benefit from voting is:

$$B_A^1 = \sum_{a_1=0}^{K-1}\sum_{l=0}^{a_1}\sum_{m=0}^{K-a_1-1}\sum_{a_2=0}^{N-K}\sum_{i=0}^{a_2}\sum_{v=0}^{a_2} P_{a_1}^{K-1}(\alpha)P_l^{a_1}(p_A^1)P_m^{K-a_1-1}(p_B^1)P_{a_2}^{N-K}(\alpha)\cdot \tag{27}$$

$$\cdot P_i^{a_2}(p_A^2(l-m))P_v^{a_2}(p_A^2(l-m+1))(X+Y+Z)),$$

where, denoting $b_2 = N - K - a_2$,

[12] As in the two-voter model analyzed in the previous section,there are cases when d takes off-equilibrium path values, and hence $\omega(i|d)$ has to be defined differently from using Bayes' rule. Since these cases would not affect equilibrium voter welfare or equilibrium candidates' winning probabilities, I do not explicitly specify the beliefs in such situations.

$$X = \sum_{j=l-m+i+1}^{b_2} P_j^{b_2}\left(p_B^2(l-m)\right)\left(\frac{1}{2}P_{l-m+v+1}^{b_2}\left(p_B^2(l-m+1)\right) + \sum_{w=0}^{l-m+v} P_w^{b_2}\left(p_B^2(l-m+1)\right)\right), \quad (28)$$

the utility from the cases when the voter's participation turns A's loss into a tie or a victory;

$$Y = \frac{1}{2}P_{l-m+i}^{b_2}\left(p_B^2(l-m)\right)\left(\sum_{w=0}^{l-m+v} P_w^{b_2}\left(p_B^2(l-m+1)\right) - \sum_{w=l-m+v+2}^{b_2} P_w^{b_2}\left(p_B^2(l-m+1)\right)\right), \quad (29)$$

the utility when the voter's participation turns A's victory into a tie or a loss.

$$Z = -\sum_{j=0}^{l-m+i-1} P_j^{b_2}\left(p_B^2(l-m)\right)\left(\frac{1}{2}P_{l-m+v+1}^{b_2}\left(p_B^2(l-m+1)\right) + \sum_{w=l-m+v+2}^{b_2} P_w^{b_2}\left(p_B^2(l-m+1)\right)\right), \quad (30)$$

the utility from the cases when the voter's participation turns A's victory into tie or loss.[13]

Likewise, one can construct the expected benefit for a first period B-supporter, B_B^1, and (2K+1) functions for the second period supporters of each candidate $B_T^2(d)$, $T \in \{A, B\}$, $d \in \{-K, .., K\}$. For example, the benefit function for a second period A-supporter conditional on the observed first period vote difference d is

$$B_A^2(d) = \frac{1}{2}\sum_{a_2=0}^{N-K-1}\sum_{l=0}^{a_2} P_{a_2}^{N-K-1}(\alpha)P_l^{a_2}\left(p_A^2(d)\right)\left(P_{l+d}^{b_2-1}\left(p_B^2(d)\right) + P_{l+d+1}^{b_2-1}\left(p_B^2(d)\right)\right). \quad (31)$$

The equilibrium is then given by the following system of 2(2K+1)+2 inequalities:

$$B_A^1 \geq c_A^1, B_B^1 \geq F^{-1}(p_B^1), \quad (32)$$

$$B_A^2(-K) \geq F^{-1}(p_A^2(-K)), \dots, B_A^2(K) \geq F^{-1}(p_A^2(K)),$$

$$B_B^2(-K) \geq F^{-1}(p_B^2(-K)), \dots, B_B^2(K) \geq F^{-1}(p_B^2(K)),$$

with equalities when $F^{-1}(p_T^t(d)) = c_T^t(d) < c_{max}$, $t \in \{1,2\}$, $T \in \{A, B\}$, $d \in \{-K, .., K\}$.

[13] Technically, formulas (19)–(22) can be obtained from the corresponding formulas (13)–(16) by replacing $p_T^t(l+m)$ with$p_T^t(l-m)$. This is because the only difference in the expressions for a first period voter's expected benefit from voting under partial and full information regimes is in the information available to second period voters. Under partial information, itis the total number of A-supporters (l) and B-supporters (m) who cast votes, i.e. $l+m$, while under full information, it is both l and m (however, what matters for a second period voter is not the numbers but their difference, $l-m$).

4.2 Welfare and Winning Probabilities

Under any information regime, candidate A wins an election whenever the number of participating A-supporters is strictly higher than the number of participating B supporters, or whenever their numbers are equal and the coin flip favors A. Therefore, given voters' equilibrium strategies, the probability that candidate A will win the election under each information regime is given by:

$$W_A^N = \sum_{a=0}^{N} \sum_{l=0}^{a} P_a^N(\alpha) P_l^a(p_A) \left(\sum_{m=0}^{l-1} P_m^{N-a}(p_B) + \frac{1}{2} P_l^{N-a}(p_B) \right), \tag{33}$$

$$W_A^P = \sum_{a_1=0}^{K} \sum_{l=0}^{a_1} \sum_{m=0}^{K-a_1} \sum_{a_2=0}^{N-K} \sum_{i=0}^{a_2} P_{a_1}^K(\alpha) P_l^{a_1}(p_A^1) P_m^{K-a_1}(p_B^1) P_{a_2}^{N-K}(\alpha) \cdot \tag{34}$$

$$\cdot P_i^{a_2}(p_A^2(l+m)) \left(\sum_{v=0}^{l-m+i-1} P_v^{N-K-a_2}(p_B^2(l+m)) + \frac{1}{2} P_{l-m+i}^{N-K-a_2}(p_B^2(l+m)) \right),$$

$$W_A^F = \sum_{a_1=0}^{K} \sum_{l=0}^{a_1} \sum_{m=0}^{K-a_1} \sum_{a_2=0}^{N-K} \sum_{i=0}^{a_2} P_{a_1}^K(\alpha) P_l^{a_1}(p_A^1) P_m^{K-a_1}(p_B^1) P_{a_2}^{N-K}(\alpha) \cdot \tag{35}$$

$$\cdot P_i^{a_2}(p_A^2(l-m)) \left(\sum_{v=0}^{l-m+i-1} P_v^{N-K-a_2}(p_B^2(l-m)) + \frac{1}{2} P_{l-m+i}^{N-K-a_2}(p_B^2(l-m)) \right).$$

Finally, average expected voter welfare is given by:

$$EW^R = \frac{1}{N}\Big(K\big(\alpha EW_{A,1}^R + (1-\alpha)EW_{B,1}^R\big) + (N-K)\big(\alpha EW_{A,2}^R + (1-\alpha)EW_{B,2}^R\big)\Big), \tag{36}$$

$R \in \{N, P, F\}$, where $EW_{T,t}^R$ is the expected welfare under regime R of a voter who supports candidate $T \in \{A, B\}$ and acts in period $t \in \{1,2\}$. Note that for any voter, EW can be expressed as follows:

$$EW = \int_{c_{min}}^{c^*} (u-c)\,\mathrm{d}F(c) + \int_{c^*}^{c_{max}} v\,\mathrm{d}F(c) = v + p^*(u-v) - \int_{c_{min}}^{F^{-1}(p^*)} c\,\mathrm{d}F(c) \tag{37}$$

where v is the probability that the voter's preferred candidate will win (i.e. voter's expected benefit) if the voter abstains; u is the probability that the voter's preferred candidate will win if he or she participates; p^* is the voter's participation probability; and $c^* = F^{-1}(p^*)$ is the corresponding cost threshold. For example, under the F-regime for a first round A-type voter, v is given by a formula that can be obtained by substituting N with $N-1$ and K with $K-1$ in formula (35):

$$v_{A,1}^F = \sum_{a_1=0}^{K-1}\sum_{l=0}^{a_1}\sum_{m=0}^{K-a_1-1}\sum_{a_2=0}^{N-K}\sum_{i=0}^{a_2} P_{a_1}^{K-1}(\alpha)P_l^{a_1}(p_A^1)P_m^{K-a_1-1}(p_B^1)P_{a_2}^{N-K}(\alpha)\cdot \quad (38)$$

$$\cdot P_i^{a_2}(p_A^2(l-m))\left(\sum_{v=0}^{l-m+i-1} P_v^{N-K-a_2}(p_B^2(l-m)) + \frac{1}{2}P_{l-m+i}^{N-K-a_2}(p_B^2(l-m))\right).$$

In other words, v in this case is the probability that candidate A will win if there are $N-1$ other potential voters in elections $K-1$ who act early. Likewise, u can be expressed as:

$$u_{A,1}^F = \sum_{a_1=0}^{K-1}\sum_{l=0}^{a_1}\sum_{m=0}^{K-a_1-1}\sum_{a_2=0}^{N-K}\sum_{i=0}^{a_2} P_{a_1}^{K-1}(\alpha)P_l^{a_1}(p_A^1)P_m^{K-a_1-1}(p_B^1)P_{a_2}^{N-K}(\alpha)\cdot \quad (39)$$

$$\cdot P_i^{a_2}(p_A^2(l-m+1))\left(\sum_{v=0}^{l-m+i} P_v^{N-K-a_2}(p_B^2(l-m)) + \frac{1}{2}P_{l-m+1+i}^{N-K-a_2}(p_B^2(l-m+1))\right).$$

That is, u is the probability that candidate A will win if there are $N-1$ other potential voters in elections $K-1$ who vote early, and there is already one vote cast for A. Similarly, one can construct the expected welfare for all other voter types and information regimes.

4.3 Some General Results

Unfortunately, obtaining a closed-form solution or even characterizing equilibria for the general model is not possible. Nevertheless, a number of general results can be stated. First, equilibrium always exists in this model. For example, in case of full disclosure, one can define a function $L: [c_{min}, c_{max}]^{2(2K+1)+2} \to [c_{min}, c_{max}]^{2(2K+1)+2}$ as:

$$L(c_A^1, c_B^1, c_A^2(-K), \ldots, c_A^2(K), c_B^2(-K), \ldots, c_B^2(K)) = \quad (40)$$
$$= (max\{min\{B_A^1, c_{max}\}, c_{min}\}, max\{min\{B_B^1(c_A, c_B), c_{max}\}, c_{min}\},$$
$$max\{min\{B_A^2(-K), c_{max}\}, c_{min}\}, \ldots, max\{min\{B_A^2(K), c_{max}\}, c_{min}\},$$
$$max\{min\{B_B^2(-K), c_{max}\}, c_{min}\}, \ldots, max\{min\{B_B^2(K), c_{max}\}, c_{min}\}).$$

Then Brouwer's fixed point theorem will imply the existence of equilibrium. Likewise, equilibrium exists under no disclosure[14] and partial disclosure regimes.

Indeed, such an equilibrium is generally not unique. For example, under full disclosure, there are always equilibria in which none of the later voters participates for some or all $|d| > 1$, $d \in \{-K, .., K\}$. More precisely, it is always an equilibrium when later (second period) A supporters abstain for all $-K \leq d < -1$ and $1 \leq d \leq K$ and later B supporters abstain for all $-K \leq d \leq -1$ and $1 < d \leq K$. Clearly, if the vote difference after the first period is more than 1, then there is always an equilibrium when all the second period voters abstain, since the deviation of one voter is never profitable to him/her. A situation in which all second period voters abstain for some subset of such values of d is also part of some equilibria. Further note that, under the F-regime, if $|d| > N - K$, that is, the number of second period voters is smaller than the vote difference after the first round, full abstention for such d must be a part of any equilibrium, since none of the second round voters can ever be pivotal.

From this point, I will focus on equilibria with strictly positive participation for all $|d| \leq N - K$ under the F-regime whenever such equilibria exist. Although neither conditions for the existence nor for the uniqueness of such equilibria can be derived analytically, the numerical simulations consistently show that there is unique equilibrium with strictly positive participation for all $|d| \leq N - K$, $d \in \{-K, .., K\}$.[15]

In the case of the N-regime, which is equivalent to simultaneous voting, uniqueness of equilibrium also cannot be generally guaranteed. Borgers (2004) proves the uniqueness of the symmetric equilibrium in the case when $\alpha = 0.5$, i.e. when the candidates have ex-ante equal support. Krasa and Polborn (2010) derive conditions sufficient to guarantee equilibrium

[14] The existence of equilibrium under simultaneous voting, which is equivalent to a no disclosure regime, is illustrated in a similar way in Krasa and Polborn (2009).

[15] Indeed, if c_{min} is sufficiently high, full abstention can be the only equilibrium. For the simulations, I let c_{min} be zero to avoid such trivial cases.

uniqueness for arbitrary α, but acknowledge that they cannot prove uniqueness for a general cost distribution. However, they argue that inall their numerical examples the equilibrium appears to be unique. The results of my simulations are fully consistent with these findings.

Finally, note that when α takes one of the extreme values, 0 or 1, a partial disclosure regime must be equivalent to full disclosure on the equilibrium path in all dimensions. This is because when a second round voter knows with certainty that any other voter is either an A-supporter ($\alpha = 1$) or a B-supporter ($\alpha = 0$), observing the first round turnout is equivalent to observing actual votes, since all first round participants must have cast their votes for one candidate. Since voters' equilibrium participation rules must be continuous functions of candidates' ex-ante support α, this equivalence implies that whenever there is a difference in any characteristic of an equilibrium under full disclosure and partial disclosure, it must reduce towards the extreme values of α.

4.4 Numerical Solution

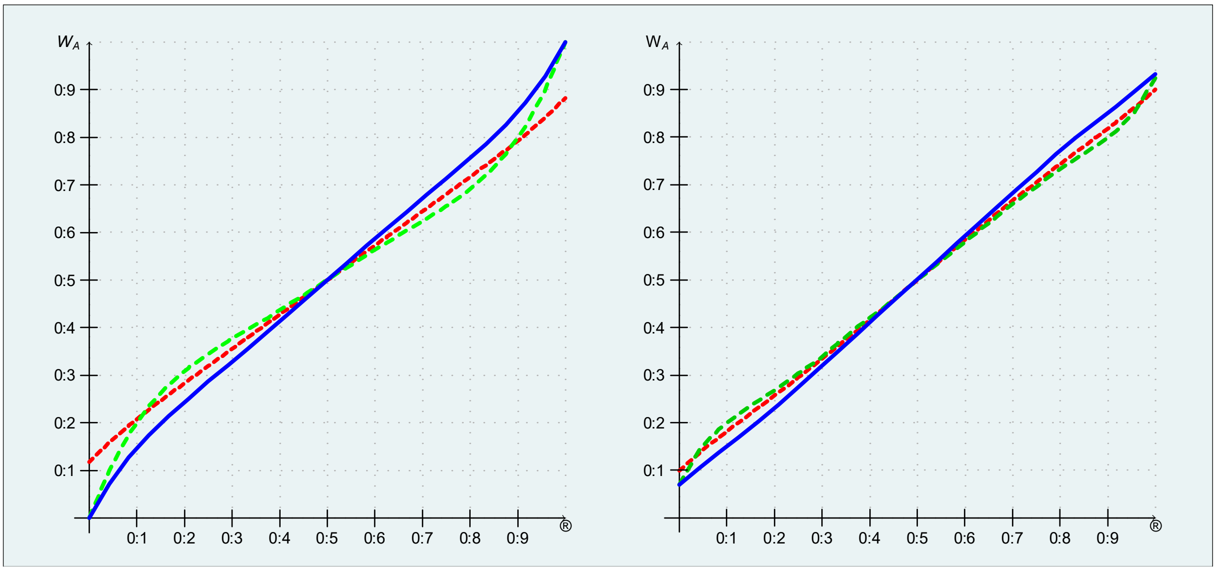
In this section, I present the results of numerical simulations of the general model. The simulations provide a number of consistent observations about equilibria and their properties. I simulate the general model for several sets of parameters. Specifically, I assume that costs are distributed uniformly over the interval $[0,0.5]$, as in the case of the two-voter model, and consider different total numbers of voters as well as their distribution between the two rounds of voting: $(N = 3, K = 2)$; $(N = 5, K = 3)$; $(N = 10, K = 7)$; $(N = 20, K = 15)$; $(N = 20, K = 10)$; $(N = 20, K = 5)$. For each set of parameters, I simulate the model going over 101 values of α from 0 to 1 at a 0.01 step interval. As a result, for each of the three information disclosure regimes, I calculate voter equilibrium participation, ex-ante expected voter welfare, and candidates' probabilities of winning as functions of α. For the sake of space, I present the results only for the first two parameters sets. The results for the rest are fully consistent with those of the two sets presented, and also with the results of the two-voter model, and are available upon request.

Candidates' Winning Probabilities

Figure 3 illustrates the evolutions of candidate A's winning probability when the total number of voters increases. The results are fully consistent with the result of the two-voter model. First, in all the cases, when $\alpha > 0.5$, i.e. candidate A has the ex-ante majority support, full disclosure gives him/her the highest probability of winning, while when $\alpha < 0.5$, full disclosure is the least beneficial regime for A.

Second, when the candidate is in the minority but his/her support is above a certain level, i.e. $\alpha^* < \alpha < 0.5$, partial disclosure is the best from the candidate's perspective, while when $\alpha < \alpha^*$ and approaches 0, no disclosure is the most preferred regime. An important result is that α^*, the critical value of α beyond which A has a greater chance of winning under the P-regime than under the N-regime, is decreasing with larger numbers of voters, and quickly approaches 0 with quite modest values of N.[16]

Finally, with larger numbers of voters, the probabilities of winning under partial disclosure and under no disclosure converge to each other but not to the probability under full information. The intuition behind this result is that, with larger numbers of voters, observing turnout only allows second period voters to make less precise estimates of the difference in vote count after the first period between two candidates, thus making it less informative.



(a) $N = 3, K = 2$ (b) $N = 5, K = 3$

Figure 3: A's winning probability: No information (dotted), partial information (dashed), full information (solid)

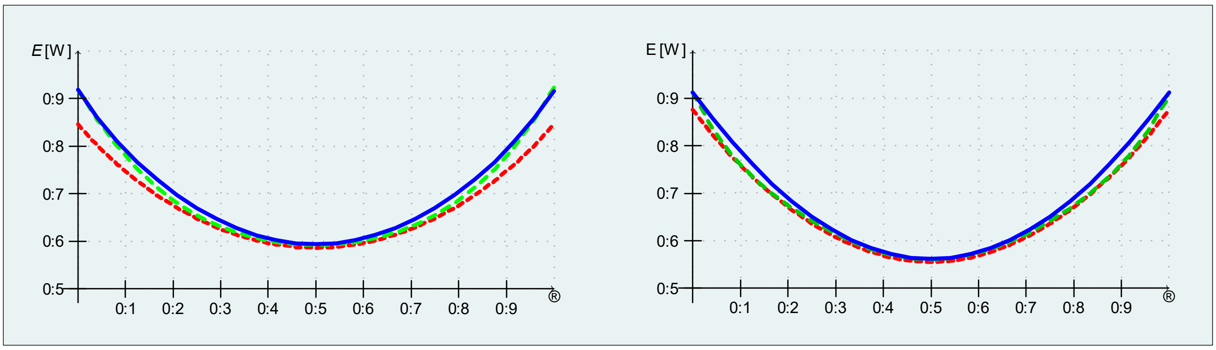
[16] As discussed in the analysis of the two-voter model, α^* never reaches 0, since at $\alpha = 0$ the characteristics of the equilibrium, including A's probability of winning, under partial disclosure must be equivalent to those under full disclosure.

Voter Welfare

Figure 8 illustrate ex-ante expected voter welfare under three information regimes as functions of α. The welfare results from the general model are somewhat different from the results from the two-voter model due to the disappearance of the corner solution for the partial information regime that existed in the general model. First, full disclosure consistently delivers higher welfare than the other two regimes.

Second, for already very moderate numbers of voters, the difference between partial and no disclosure disappears almost everywhere except when α is close to its extreme values, 0 and 1. The reason is that, with more voters, the informativeness of turnout for moderate values of α quickly decreases.

As discussed above, when α approaches extreme values, partial disclosure converges to full disclosure.



(a) $N = 3, K = 2$ (b) $N = 5, K = 3$

Figure 4: Average expected voter welfare: No information (dotted), partial information (dashed), full information (solid)

5 Discussion and Concluding Remarks

This study was motivated by observing real policies governing information disclosure on the actions of early voters in elections. In this paper, I theoretically study the effect of information about the actions of early voters in elections with costly sequential participation on voter and candidate welfare. Using a two-voter pivotal costly voting model and numerical simulations of a general model, I assess the effects of three information regimes: no disclosure, partial (turnout only) disclosure and full (votes) disclosure, and consistently obtain the following key results:

1) Full disclosure dominates the other two regimes in terms of voter welfare.

2) An ex-ante majority candidate has the greatest chance to win under full disclosure, a minority candidate with relatively high support – under partial disclosure, and a minority candidate with low support – under no disclosure.

3) Both voter and candidate welfare under partial and no disclosure regimes quickly converge when there are larger numbers of voters.

The first finding has a straightforward implication, suggesting that welfare maximizing policy makers should always allow for full information disclosure during election days. The second finding suggests that the way an information regime choice affects candidates' chances to win elections depends on the candidates' ex-ante support. Since it is often the case that an incumbent candidate can set the rules of the game by choosing an information regime, his/her specific choice may then signal his/her support expectations.

While these findings are robust, they nevertheless should be interpreted with care, especially in a discussion of their applicability to real life elections. First, numerically, the differences between outcomes under different information regimes are relatively small[17], especially for larger numbers of voters. Second, pivotal costly voting models, similar to the one used in the paper, are generally known to work better in small elections settings including voting in organizations, committees, etc., where the number of voters is small, than in large election settings such as national elections.[18] Therefore, this paper is not intended to advocate for

[17] However, the differences in voter welfare are not as negligible as they may seem from the graphs even for relatively large numbers of voters, since voter welfare is presented in terms of expected per voter utility.

[18] The main critique of pivotal voting models is their inability to replicate high participation rates observed in large real life elections: individual pivotal probabilities rapidly decrease with larger numbers of voters, and hence the

immediate changes in electoral legislation. Instead, it aims to bring attention to the fact that electoral laws regulating the availability of information to voters on election days may impact both election outcomes and welfare, to suggest a potential mechanism underlying these impacts, and to encourage more research on information disclosure in elections, particularly in the context of large elections in which the welfare stakes are high.

equilibrium participation rates also decrease. Nevertheless, though voters' participation decisions in large elections are unlikely to be exclusively driven by pivotal perspectives, there is substantial evidence suggesting that even in large elections voters care about being pivotal to a large extent. See, for example, Cox and Munger (1989), Shacharand and Nalebuff (1999), Fauvalle-Aymar and Francois (2006), De Paola and Scoppa (2014).

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Appendix

A Candidate Welfare

Candidate A wins when 1) both voters are A-supporters except in the case when none of them votes and the coin flip favors B; 2) both voters are B-supporters, none of them votes and the coin flip favors A; 3) one voter supports A, one voter supports B, andonly the A-supporter votes; 4) one voter supports A, one voter supports B, either both or neither of them votes, and the coin flip favors A.

No disclosure:

$$W_A^N = \alpha^2\left(1-\frac{1}{2}(1-p_A^1)(1-p_A^2)\right)+\frac{1}{2}(1-\alpha)^2(1-p_B^1)(1-p_B^2)+\alpha(1-\alpha)(p_A^1(1-p_B^2)+(1-p_B^1)p_A^2))+$$
$$+\frac{1}{2}\alpha(1-\alpha)\left((1-p_A^1)(1-p_B^2)+(1-p_B^1)(1-p_A^2)+p_A^1p_B^2+p_B^1p_A^2\right), \quad \text{(A.1)}$$

where p_T^t, $T\in\{A,B\}$, $t\in\{1,2\}$, are the equilibrium participation probabilities (2). Simplifying the expression gives:

$$W_A^N = \frac{1}{2}+\frac{3}{2}\frac{2\alpha-1}{(2-\alpha)^2(1+\alpha)^2}. \quad \text{(A.2)}$$

Partial disclosure:

$$W_A^P = \alpha^2\left(1-\frac{1}{2}(1-p_A^1)(1-p_A^2(0))\right)+\frac{1}{2}(1-\alpha)^2(1-p_B^1)(1-p_B^2(0))+$$
$$+\alpha(1-\alpha)(p_A^1(1-p_B^2(1))+(1-p_B^1)p_A^2(0)))+$$
$$+\frac{1}{2}\alpha(1-\alpha)\left((1-p_A^1)(1-p_B^2(0))+(1-p_B^1)(1-p_A^2(0))+p_A^1p_B^2(1)+p_B^1p_A^2(1)\right). \quad \text{(A.3)}$$

Plugging the equilibrium strategies (7):

$$W_A^P = \alpha+\frac{1}{2}\alpha(1-\alpha)((2-\gamma)p_A^1-(1+\gamma)p_B^1), \quad \text{(A.4)}$$

where γ is given by formula (8), and p_A^1 and p_B^1 are the equilibrium strategies (6).

Full disclosure:

$$W_A^F = \alpha^2\left(1-\frac{1}{2}(1-p_A^1)(1-p_A^2(0))\right)+\frac{1}{2}(1-\alpha)^2(1-p_B^1)(1-p_B^2(0))+$$
$$+\alpha(1-\alpha)(p_A^1(1-p_B^2(+))+(1-p_B^1)p_A^2(0)))+$$
$$+\frac{1}{2}\alpha(1-\alpha)\left((1-p_A^1)(1-p_B^2(0))+(1-p_B^1)(1-p_A^2(0))+p_A^1p_B^2(+)+p_B^1p_A^2(-)\right). \quad \text{(A.5)}$$

Re-arranging the terms:

$$W_A^F = \alpha^2+\alpha(1-\alpha)(1-p_B^1)+\frac{1}{2}\alpha(1-\alpha)(p_A^1p_B^2(+)+p_B^1p_A^2(-)). \quad \text{(A.6)}$$

Plugging the equilibrium participation probabilities (9):

$$W_A^F = \frac{1}{2}\alpha\left(1+\alpha+\frac{(1-\alpha)^2}{2}\right). \quad \text{(A.7)}$$

B Effect of α on Candidate Welfare

Consider candidate welfare under an N-regime as a function of α and participation probabilities, given by formula (A.1). Differentiating the expression above with respect to α, plugging the equilibrium values for p_A^1, p_A^2, p_B^1 and p_B^2 given by formulas (2), and evaluating the resulting expression at $\alpha=\frac{1}{2}$, one may obtain the “support effect”:

$$\left.\frac{\partial W_A^N}{\partial\alpha}\right|_{p_A=\frac{1}{1+\alpha},p_B=\frac{1}{2-\alpha},\alpha=\frac{1}{2}} = \frac{8}{9}. \quad \text{(A.8)}$$

Plugging the equilibrium values for p_A^1, p_A^2, p_B^1 and p_B^2 into expression (A.1) first (the result is given by expression (2), then differentiating it with respect to α, and then evaluating the resulting expression at$\alpha=\frac{1}{2}$, one may obtain the total effect, which is “support effect” plus “participation effect”:

$$\left.\frac{\partial W_A^N}{\partial\alpha}\right|_{\alpha=\frac{1}{2}} = \left.\frac{\partial\left(\frac{1}{2}+\frac{3}{2}\frac{2\alpha-1}{(2-\alpha)^2(1+\alpha)^2}\right)}{\partial\alpha}\right|_{\alpha=\frac{1}{2}} = frac1627. \quad \text{(A.9)}$$

The difference between expressions (A.14) and (A.13) is “participation effect”: $\frac{16}{27}-\frac{8}{9}=-\frac{8}{27}$.

To evaluate the effect of α on candidate welfare under a P-regime at $\alpha = 0.5$, one may plug $\gamma = 0.5$ into expression (3) to obtain

$$W_A^P = \alpha + \frac{3}{4}\alpha(1-\alpha)(p_A^1 - p_B^1). \quad \text{(A.10)}$$

Differentiating the expression above with respect to α, plugging the equilibrium values for p_A^1 and p_B^1 given by formulas (6), and evaluating the resulting expression at $\alpha = \frac{1}{2}$, one may obtain the “support effect”:

$$\left.\frac{\partial W_A^P}{\partial \alpha}\right|_{p_A^1 = \frac{3}{2}(1-\alpha), p_B^1 = \frac{3}{2}\alpha, \alpha = \frac{1}{2}} = 1. \quad \text{(A.11)}$$

Plugging the equilibrium values for p_A^1 and p_B^2 into expression (A.15) first, then differentiating it with respect to α, and then evaluating the resulting expression at $\alpha = \frac{1}{2}$, one may obtain the total effect, which is “support effect” plus “participation effect”:

$$\left.\frac{\partial W_A^P}{\partial \alpha}\right|_{\alpha = \frac{1}{2}} = \left.\frac{\partial\left(\alpha + \frac{9}{8}\alpha(1-\alpha)(1-2\alpha)\right)}{\partial \alpha}\right|_{\alpha = \frac{1}{2}} = \frac{7}{16}. \quad \text{(A.12)}$$

The difference between expressions (A.17) and (A.16) is “participation effect”: $\frac{7}{16} - 1 = -\frac{9}{16}$.

C Voter Welfare

If a voter abstains, his/her preferred candidate wins when: 1) the other voter supports the same candidate and votes; 2) the other voter abstains and the coin flip favors the preferred candidate. If a voter participates, his/her candidate always wins except when the other voter supports the opposing candidate, votes, and the coin flip favors the opposing candidate. Therefore, under **no information**:

$$v_A^1 = v_A^2 = \alpha p_A + \frac{1}{2}(\alpha(1-p_A) + (1-\alpha)(1-p_B)). \quad \text{(A.13)}$$

$$u_A^1 = u_A^2 = 1 - \frac{1}{2}(1-\alpha)p_B. \quad \text{(A.14)}$$

Substituting these expressions into formula (6), plugging equilibrium values for p_A and p_B, and re-arranging the terms, one may obtain:

$$EW_A^1 = EW_A^2 = \frac{1}{4}\left(1 + \frac{\alpha^2}{(1+\alpha)^2} + \frac{2}{2-\alpha}\right). \quad \text{(A.15)}$$

The expression for EW_B can be obtained by substituting α with $1-\alpha$ in the above formula:

$$EW_B^1 = EW_B^2 = \frac{1}{4}\left(1 + \left(\frac{1-\alpha}{2-\alpha}\right)^2 + \frac{2}{1+\alpha}\right). \quad \text{(A.16)}$$

Under **partial information** disclosure, a second period voter obtains utility 1 minus the expected cost 0.25 if he or she observes no participation in the first period regardless of the candidate he or she supports, i.e. $EW_A^2(0) = EW_B^2(0) = 0.75$.If he or she supports A and observes that the first period voter participated, his/her expected benefit if he or she abstains is $v_A^2(1) = \gamma$; and his/her expected benefit if he or she votes is $u_A^2(1) = \gamma + 0.5(1-\gamma)$. Plugging these valuestogether with equilibrium participation $p_A^2(1) = 1-\gamma$ into formula (6), his/her expected utility is:

$$EW_A^2(1) = \gamma + \frac{1}{2}(1-\gamma)^2 - \frac{1}{4}(1-\gamma)^2 = \gamma + \frac{1}{4}(1-\gamma)^2. \quad \text{(A.17)}$$

Likewise:

$$EW_B^2(1) = 1 - \gamma + \frac{1}{4}\gamma^2. \quad \text{(A.18)}$$

Given the first-round equilibrium participation probabilities (6), the ex-ante probability of observing positive participation after the first period is

$$P(1) = \begin{cases} \alpha + (1-\alpha)\alpha(1+\gamma), \alpha < \frac{1}{3}, \\ 3\alpha(1-\alpha), \alpha \in [\frac{1}{3}; \frac{2}{3}], \\ 1 - \alpha + \alpha(1-\alpha)(2-\gamma), \alpha > \frac{2}{3}. \end{cases} \quad \text{(A.19)}$$

For $\alpha \in [1/3, 2/3]$:

$$v_A^1 = \alpha p_A^2(0) + \frac{1}{2}(\alpha(1-p_A^2(0)) + (1-\alpha)(1-p_B^2(0))) = \alpha. \quad \text{(A.20)}$$

$$u_A^1 = 1 - \frac{1}{2}(1-\alpha)p_B^2(1) = \frac{3}{4} - \frac{1}{4}\alpha. \quad \text{(A.21)}$$

Plugging the above expressions into formula (6):

$$EW_A^1 = \alpha + \frac{9}{16}(1-\alpha)^2. \quad \text{(A.22)}$$

Likewise,

$$EW_B^1 = 1 - \alpha + \frac{9}{16}\alpha^2. \quad \text{(A.23)}$$

Therefore, the total average voters' expected welfare under partial information is

$$EW^P = \frac{1}{2}(\alpha EW_A^1 + (1-\alpha)EW_B^1 + EW^2(0)(1-3\alpha(1-\alpha)) + EW^2(1)3\alpha(1-\alpha)) = \frac{7}{8} - \alpha(1-\alpha). \quad \text{(A.24)}$$

Following the same logic, one may derive welfare for $\alpha < 1/3$:

$$EW^P = \frac{\alpha}{2}\left(\frac{3}{4} - \frac{(1-\alpha)\gamma}{2}\right) + \frac{1-\alpha}{2}\left(1 - \alpha + \frac{\alpha^2(1-\gamma^2)}{2(1-\alpha)\gamma} - \frac{\alpha^2(1-\gamma)^2}{4(1-\alpha)^2\gamma^2}\right) + \frac{3}{4}\left(1 - \frac{\alpha}{\gamma}\right) +$$
$$+ \frac{\alpha}{\gamma}\left(\alpha\left(\gamma + \frac{(1-\gamma)^2}{4}\right) + (1-\alpha)\left(1 - \gamma + \frac{\gamma^2}{4}\right)\right), \quad \text{(A.25)}$$

where γ is the posterior probability that candidate A will lead after the first period conditional on the observed turnout being 1 given by formula (8). Similarly, for $\alpha > 2/3$ the expression for voters' welfare may be obtained by replacing α with $1 - \alpha$ in the above formula.

Under **full information**, the first period A supporter's expected utility is α if he or she abstains, and $1 - 0.25\alpha(1-\alpha)$ if he or she participates, since in the latter case A loses only when the second period voter supports B, participates and the coin flip favors B. Then, from formula (6):

$$EW_A^1 = \alpha + \frac{1}{2}(1-\alpha)^2 - \frac{1}{4}(1-\alpha)^2 = \alpha + \frac{1}{4}(1-\alpha)^2. \quad \text{(A.26)}$$

$$EW_B^1 = 1 - \alpha + \frac{1}{4}\alpha^2. \quad \text{(A.27)}$$

If a second period A-supporter observes abstention, his/her expected utility is 1 minus the expected cost 0.25. If he or she observes A leading after the first period, his/her expected utility is 1. If he or she observes A losing, his/her expected utility is 0.25. Weighting these utilities by the ex-ante probabilities of observing each possible outcome of the first period, one may obtain:

$$EW_A^2 = \frac{3}{4}(\alpha^2 + (1-\alpha)^2) + \alpha(1-\alpha) + \frac{1}{4}\alpha(1-\alpha) = \frac{3}{4} - \frac{1}{4}\alpha(1-\alpha). \quad \text{(A.28)}$$

The expression for EW_B^2 is exactly the same.

The voters' average expected welfare is then

$$EW^F = \frac{1}{2}\left(\alpha\left(\alpha + \frac{1}{4}(1-\alpha)^2\right) + (1-\alpha)\left(1-\alpha+\frac{1}{4}\alpha^2\right) + \frac{3}{4} - \frac{1}{4}\alpha(1-\alpha)\right) = \frac{7}{8} - \alpha(1-\alpha). \quad \text{(A.29)}$$